University of Limerick

Ollscoil Luimnigh

College of Informatics and Electronics

END–OF–TERM ASSESSMENT

MODULE CODE:	MA4002	DURATION OF EXAM:	$2\frac{1}{2}$ hours
MODULE TITLE:	Engineering Maths 2	FRACTION OF TOTAL MARKS:	100%
TERM:	Spring 1996	LECTURER:	Dr. E. Gath

INSTRUCTIONS TO CANDIDATES: Answer question **1** and any three of **2, 3, 4, 5, 6, 7**.

1.(i) [4] A car has acceleration $a(t) = 2 + 15\sqrt{t}$ at time t. It starts from rest, at time $t = 0$, from position $s = 0$. Determine its velocity and position at all times $t \geq 0$.

(ii) [4] Evaluate the indefinite integral $\int \sqrt{x} \ln x \, dx$.

(iii) [4] Find $\frac{dy}{dx}$ when $y = \int_0^{x^2} \tan t \, dt$.

(iv) [4] Find the average value of $y = \frac{1}{x}$ on the interval $1 \leq x \leq 4$.

(v) [4] Sketch the output which results from implementing the MapleVR3 command:

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with(student):
leftbox(x^2, x=0..1, 4);
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(vi) [4] Express as a definite integral, and hence evaluate, the limit of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} (\cos c_i) \, \Delta x_i$, where P is the partition with $x_i = \frac{2i}{n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$.

(vii) [4] The quantities x and y are measured with relative error e_x and e_y respectively. The quantity Q is then calculated from the formula $Q = \frac{x^3}{y+1}$.
Find the relative error e_Q in terms of e_x, e_y, x and y.

(viii) [4] Solve the initial value problem $\frac{dy}{dx} = (1 + y^2) \cos x$, with $y(0) = 0$.

(ix) [4] Use the *Euler method* to write down an iterative approximation of the solution of the initial value problem $y' = x^2 + y^2$, $y(0) = 2$, choosing step size $h = 0.1$.

(x) [4] Evaluate the determinant $\begin{vmatrix} 1 & 4 & 7 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{vmatrix}$.

2.[6+7+7] Evaluate each of the integrals:

(i) $\displaystyle\int \sec x\,dx$ (ii) $\displaystyle\int_0^\infty \frac{1}{x^2+2x+4}\,dx$ (iii) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{4-5\sin t}\,dt$

Note re (iii): $\tan\frac{\pi}{8} = \sqrt{2}-1$.

3.[20] Attempt any *three* of parts (i), (ii), (iii), (iv).

(i) Find the area enclosed between $y = \cosh x$ and $y = \sinh x$ for $x \geq 0$.

(ii) Find the volume of the solid of revolution that results from revolving the region, enclosed by the curves $y = \dfrac{3-x}{x}$, $x = 0$, $y = 0$ and $y = 2$, about the y-axis.

(iii) Find the mass of a rod with mass density $\rho(x) = \rho_0(1+x)$, for $0 \leq x \leq 4$. Find the moment of inertia of this rod when it is rotating about the point $x = 2$.

(iv) A particle has position vector $\mathbf{r}(t) = (1-3t^2)\,\mathbf{i} + (t-3t^3)\,\mathbf{j}$ at time t. Find the distance travelled by the particle between times $t = 0$ and $t = 2$.

4. (a)[10] Find the Taylor Series, up to and including quadratic terms, of $z = f(x,y) = xe^{2y} + \sin(x+y^2)$ about the point $(\frac{\pi}{2}, 0)$.

(b)[10] The least squares line approximation to the points $(0,1)$, $(1,2)$, $(2,4)$, $(3,b)$, $(4,6)$ is $y = 1 + \dfrac{11}{10}x$. Find b.

5.(a)[10] Set up an iterative reduction formula for $I_n \equiv \displaystyle\int_0^{\frac{\pi}{2}} \cos^{2n} x\,dx$, $n = 0,1,2,3,\ldots$, and hence prove that $I_n = \dfrac{(2n)!\pi}{2^{2n+1}n!n!}$.

(b)[10] Use *Simpson's rule*, with four equal subintervals (*i.e.* $n = 2$), to find an approximation for the definite integral $\displaystyle\int_0^1 \cosh(x^2)\,dx$. Assuming that $M_4 \equiv \max_{x\in[0,1]} \left|\dfrac{d^4}{dx^4}\cosh(x^2)\right| < 100$, find an upper bound for the error in the above approximation.

How many subintervals would be required to ensure accuracy to 20 decimal places?

6. The charge $q(t)$, at time t, in the capacitor in the LRC circuit depicted below satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t).$$

Consider a circuit with an inductor with $L = 1$ Henry, a resistor with $R = 4$ ohms and a capacitor with $C = 0.2$ Farads.

(a)[8] Write down the general solution to the homogeneous equation *i.e.* when the external voltage $E(t) = 0$.

(b)[8] By first finding a particular solution, find the general solution to the differential equation when the external voltage $E(t) = 8\cos t$.

(c)[4] Solve the equation in **(b)** when the initial charge on the capacitor is $q(0) = 1$ Farad and the initial current is $q'(0) = 0$ amps.

7. (a)[9] Write down a system of three linear equations in two unknowns

(i) which is inconsistent;

(ii) which has a unique solution;

(iii) which has an infinite number of solutions.

(b)[11] Find the inverse of the 3×3 matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 3 & 0 \\ 5 & 1 & 3 \end{bmatrix}.$$