University of Limerick

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END–OF–TERM ASSESSMENT

MODULE CODE:	MA4002	DURATION OF EXAM:	$2\frac{1}{2}$hours
MODULE TITLE:	Engineering Maths 2	FRACTION OF TOTAL MARKS:	100%
TERM:	Spring 1997	LECTURER:	Dr. E. Gath

INSTRUCTIONS TO CANDIDATES: Answer question **1** and any three of **2, 3, 4, 5, 6, 7**.

1.(i)[4] A car has acceleration $a(t) = 6e^{-2t}$ at time t. It starts from rest, at time $t = 0$. Find its velocity for all $t \geq 0$, and determine its *average* velocity between $t = 0$ and $t = 3$.

(ii)[4] Evaluate the indefinite integral $\int \frac{1}{x \ln x}\, dx$ and give two reasons why $\int_0^\infty \frac{1}{x \ln x}\, dx$ does not exist.

(iii)[4] Evaluate the limit $\lim_{x \to 2} \frac{1}{x-2} \int_2^x \tan t\, dt$.

(iv)[4] Give the numerical output which results from implementing the Maple command:

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with(student):
evalf(rightsum(x, x=0..3, 3));
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(v)[4] Write down, but do not evaluate, the Riemann sum corresponding to the definite integral $\int_0^3 \sin x\, dx$, taking the partition P with $x_i = \frac{3i}{n}$, for $i = 0, 1, \ldots, n$, and choosing $c_i = x_{i-1}$ (the leftmost point of each subinterval).

(vi)[4] Find the volume of the solid of revolution that results from revolving the bounded region enclosed between the curve $y = 1 - x^2$ and the x-axis, about the x-axis.

(vii)[4] Prove that $I_n \equiv \int_0^\infty x^{2n} e^{-x^2}\, dx$ satisfies $I_n = \left(\frac{2n-1}{2}\right) I_{n-1}$ for all $n = 1, 2, 3, \ldots$.

(viii)[4] The quantities x and y are measured with relative error e_x and e_y respectively. The quantity Q is then calculated from the formula $Q = x^2 \sin y$. Find the relative error e_Q in terms of e_x, e_y, x and y.

(ix)[4] Solve the initial value problem $\frac{dy}{dx} = \frac{y^2+1}{2xy}$, with $y(2) = 1$.

(x)[4] Evaluate the determinant $\begin{vmatrix} 1 & a & b \\ 2 & 2a & c \\ 3 & 3a & d \end{vmatrix}$ for any a, b, c, d.

2.[7+6+7] Evaluate each of the integrals:

(i) $\displaystyle\int e^{2x}\cos 3x\,dx$ (ii) $\displaystyle\int \frac{x^3+x^2}{x^2-4}\,dx$ (iii) $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

3.[20] Attempt any *three* of parts (i), (ii), (iii), (iv).
(i) Find the area enclosed between the circle $x^2+y^2=25$ and the line $x=3$, for $x\geq 3$.

(ii) Find the volume of the solid of revolution that results from revolving about the y-axis, the region enclosed between the curve $y=10-x-x^3$ and the x-axis, for $0\leq x\leq 2$.

(iii) Find the arc-length along the curve $y=\dfrac{x^3}{3}+\dfrac{1}{4x}$ for $1\leq x\leq 3$.

(iv) Find the mass of a rod with mass density $\rho(x)=2+\sin x$, for $0\leq x\leq \frac{\pi}{2}$. Find the moment of inertia of this rod when it is rotating about the end $x=0$.

4.(a)[10] Use *Simpson's Rule*, with four equal subintervals (*i.e.* $n=2$), to find an approximation for the definite integral $\displaystyle\int_0^\pi e^{\sin x}\,dx$. Given that $M_4\equiv \displaystyle\max_{x\in[0,\pi]}\left|\frac{d^4}{dx^4}(e^{\sin x})\right|<12$,
find an upper bound for the error in the above approximation.
How many subintervals would be required to ensure an absolute error of less than 10^{-10}?

(b)[10] Using (i) the *Euler Method* and (ii) the *Improved Euler Method*, write down an iterative scheme to solve the initial value problem $\dfrac{dy}{dx}=2y+x^2,\quad y(0)=1$.
Taking $h=0.5$, use both methods to estimate $y(1)$.

5. (a)[10] Find the Taylor Series, up to and including quadratic terms,
of $z=f(x,y)=x^2e^y+\sin(xy)$ about the point $(1,0)$.

(b)[10] It is known that the quantities y and x are related by the formula $y=kx^\alpha$, for some unknown constants k and α. By writing this as $\ln y=\alpha\ln x+\ln k$, one can use the *method of least squares* to find the best-fit line relating $\ln y$ to $\ln x$ and hence find an approximation of the constants k and α.
For the given data points, $(x,y)=(0.5,1)$, $(1,2.6)$, $(2,11)$, $(3,30)$, $(4,50)$, use this method to find an approximation of the constants k and α.

6.(a)[10] An object of mass m, projected vertically downwards with an initial velocity v_0, experiences a downward force of $mg - kv$ where g is the acceleration due to gravity and k is the drag coefficient. Applying Newton's Second Law, we find that the downward velocity v satisfies the differential equation $m\frac{dv}{dt} = mg - kv$.
Solve this differential equation to find $v(t)$ for all $t \geq 0$.
Find $\lim_{t\to\infty} v(t)$, the so called *terminal velocity.*
If the initial velocity is $v_0 = 0$, find how long it takes the object to reach 90% of the terminal velocity.

(b)[10] Solve the linear second order constant coefficient ordinary differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 4x + 1$, subject to the initial conditions $y(0) = 2$, $y'(0) = 1$.

7.(a)[5+5] Use the method of *Gauss-Jordan elimination* to find all solutions of each system of linear equations:

(i)
$$\begin{array}{rrrcr} x & +3y & +2z & = & 1 \\ 3x & +6y & +5z & = & 1 \\ x & +6y & +3z & = & 3 \end{array}$$

(ii)
$$\begin{array}{rrrcr} & y & -z & = & 3 \\ x & -y & +2z & = & 0 \\ 3x & -y & +4z & = & 1 \end{array}$$

(b)[10] Find the inverse of the 3×3 matrix

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 3 & 4 \\ 3 & -3 & 7 \end{pmatrix}.$$