UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2000

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. E. Gath

PERCENTAGE OF TOTAL MARKS: 100%

EXTERNAL EXAMINER: Prof. S. McKee

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *three* questions from questions 2, 3, 4, 5, 6 and 7. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) An object has temperature T at time t. It cools at a rate $\dfrac{dT}{dt} = -e^{-0.02t}$.
The initial temperature at time $t = 0$ is $T = 80$.
Find the temperature T at all times $t \geq 0$ and the temperature eventually reached by the object. 4%

(b) Express as a definite integral, and hence evaluate, the limit of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{1+c_i^2}\,\Delta x_i$, where P is the partition with $x_i = \dfrac{i}{n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$. 4%

(c) Evaluate $\dfrac{d}{dx}\displaystyle\int_{-\infty}^{\ln x} \dfrac{e^t}{t}\,dt$, where we assume that $0 < x < 1$. 4%

(d) Sketch the output which results from implementing the Maple command:

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with(student):
leftbox(x^2, x=-1..3, 4);
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4%

(e) State whether the given integral converges or diverges and prove your claim: $\displaystyle\int_2^{\infty} \dfrac{x^5}{x^6-1}\,dx$. 4%

(f) Find the average value of $y = x\cos x$ on the interval $0 \leq x \leq \frac{\pi}{2}$. 4%

(g) The cross-section at x, perpendicular to the x-axis, of a solid is a rectangle with sides $3x$ and $x+2$, for $0 \leq x \leq 2$. Find the area of the cross-section at x and the find the volume of the solid. 4%

(h) Find all first and second partial derivatives of
$f(x,y) = 2\cosh(\ln x - \ln y)$. (Hint: first simplify!) 4%

(i) Solve the initial value problem $\dfrac{dy}{dx} = 2x\sqrt{1-y^2}$, with $y(1) = 0$. 4%

(j) Evaluate the determinant $\begin{vmatrix} 2 & 4 & 1 \\ 1 & 3 & -2 \\ 0 & 2 & -3 \end{vmatrix}$. 4%

2 Evaluate each of the integrals:

(a) $\displaystyle\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$ 6%

(b) $\displaystyle\int_0^{\pi} e^{3x}\sin x\,dx$ 7%

(c) $\displaystyle\int \frac{x^3+4x^2}{x^2+6x+10}\,dx.$ 7%

3 Answer part (a) and *any two* of parts (b), (c), (d):

(a) Find the area of the enclosed region bounded by the curves $y = x^2 - 2x$ and $y = 12 - x^2$. 6 %

(b) The disk of radius a and centre $(b, 0)$ *i.e.* $(x-b)^2 + y^2 \leq a^2$, where $b > a > 0$, is rotated about the y-axis to form a torus. Find its volume. 7 %

(c) A particle has position vector $\mathbf{r}(t) = t^2\cos t\,\mathbf{i} + t^2\sin t\,\mathbf{j}$ at time t. Find the distance travelled by the particle between times $t = 0$ and $t = 2$. 7 %

(d) A rectangular brick has dimensions $8 \times 4 \times 2$. Find the mass and the centre of mass of the brick if the density at a point is $2 + \dfrac{x}{4}$, where x is the perpendicular distance to one of the 4×2 faces. 7 %

4 (a) Define $I_n \equiv \displaystyle\int_1^e (\ln t)^n dt,\ \ n = 0, 1, 2, \ldots.$

Prove that $I_n = e - nI_{n-1}$, for $n \geq 1$, and hence evaluate $\displaystyle\int_1^e (\ln t)^5 dt$. 7%

(b) A radioactive substance has a half-life of 1000 years. How much of an initial amount of the substance is remaining after 700 years? 6%

(c) Use the *Euler method* to write down an iterative approximation of the solution of the initial value problem $y' = x^2 + \sqrt{y},\ y(0) = 1$, choosing step size $h = 0.2$. Use the approximation to find $y(0.6)$. 7%

5 A mass of m kg attached is attached to a spring and a dashpot. The spring has Hooke constant k N/m. The displacement of the spring at time t seconds is $x(t)$ metres and its velocity $v(t) = \frac{dx}{dt}$ m/s. We assume that the dashpot exerts a damping force $-cv$ N where c is a constant. We may also subject

the mass to an external force $F(t)$ N. Applying Newton's second law of motion, the resulting equation of motion is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t).$$

(a) Consider a mass of $m = 1$ and Hooke constant $k = 4$. If the dashpot is removed and there is no forcing, find the *amplitude* of the oscillation if the initial displacement is $x(0) = 2$ and the initial velocity is $v(0) = 3$. 6%

(b) If a dashpot with constant $c = 5$ is applied to the above system find the displacement at all times $t \geq 0$, subject to the same initial conditions. 7%

(c) Assume the same values of m, k and c and suppose that an external force $F(t) = 34\cos t$ is now also applied to the mass. Find the general solution for the displacement $x(t)$ at all times $t \geq 0$. 7%

6 (a) Given that $M_4 \equiv \max_{x\in[1,3]} \left|\frac{d^4}{dx^4}e^{-\sin x}\right| < 2$, find an upper bound on the error for the *Simpson's Rule* approximation of the definite integral $\int_1^3 e^{-\sin x}\,dx$ using $2n$ subintervals. 5%

How many subintervals would be required to ensure an error of less than 10^{-6}? 1%

(b) By approximately what percentage will $w = \frac{x^2y^3}{z}$ increase/decrease if x decreases by 4%, y increases by 2% and z decreases by 5%? 4%

(c) Find the Taylor Series, up to and including quadratic terms, of $z = f(x, y) = \frac{1}{\cosh x + \sin y}$ about the point $(0, 0)$. 10%

7 (a) Find all solutions of each system of linear equations:

(i) $$\begin{array}{rrrcl} & y & +z & = & 5 \\ 3x & +2y & & = & 1 \\ 3x & & -2z & = & 2 \end{array}$$

(ii) $$\begin{array}{rrrcl} x & +4y & +5z & = & 7 \\ 2x & -3y & & = & 6 \\ 3x & -y & +z & = & 5 \end{array}$$

5%+5%

(b) Find the inverse of the 3×3 matrix 10%

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 0 & 2 \\ 5 & 4 & 1 \end{bmatrix}.$$