UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2001

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: 2 hours

LECTURER: Dr. E. Gath

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**

Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.

Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.

You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A rocket has acceleration $a = \dfrac{d^2s}{dt^2} = 20 + 30\sqrt{t}$ metres/second^2 at time t. It starts from rest at time $t = 0$. How far does it travel in the first 4 seconds? 4%

(b) Give the numerical output which results from implementing the Maple command:

```
with(student):
evalf(leftsum(xˆ2,x=0..2, 4));
```

4%

(c) State whether the given integral converges or diverges and justify your claim: $\displaystyle\int_{-1}^{1} \frac{2x+2}{x^2+2x}\,dx$. 4%

(d) Prove that $I_n \equiv \displaystyle\int_0^1 x^n e^x\,dx$ satisfies the iterative equation $I_n = e - nI_{n-1}$ for $n \geq 1$. (Hint: integrate by parts.) 4%

(e) Find all first and second partial derivatives of $f(x,y) = \cos(x^2 + y)$. 4%

(f) Using the *Improved Euler method*, with step size $h = 0.2$, write down an iterative scheme which approximates the solution of the initial value problem $y' = x + \ln(y), \quad y(0) = 1$. 4%

(g) Find the general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 3$. 4%

(h) For which value of β has the system of equations below an infinite number of solutions?

$$\begin{array}{rrrcl} x & +y & +\beta z & = & 3 \\ x & & +3z & = & 1 \\ 2x & -y & & = & 0 \end{array}$$

4%

2 Answer part (a) and *any two* of parts (b), (c), (d):

(a) Find the area of the region between $y = \dfrac{1}{4x^2+9}$ and the x-axis, for $x \geq 0$. 5%

(b) A solid of revolution is generated by revolving about the x-axis the area bounded between $x = 5y^3 + 3y$ and the y-axis for $0 \leq y \leq 1$. Find the volume of the solid so obtained. (Hint: use cylindrical shells.) 7%

(c) Find the arc-length along the curve $y = \dfrac{x^2}{8} - \ln x$ for $1 \leq x \leq 2$. 7 %

(d) Find the mass and the centre of mass of a rod with mass density $\rho(x) = 1 + \cos x$, for $0 \leq x \leq \frac{\pi}{2}$. 7%

3 The charge $q(t)$, at time t, in the capacitor in the LRC circuit depicted below satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t).$$

Consider a circuit with an inductor with $L = 1$ Henry, a resistor with $R = 6$ ohms and a capacitor with $C = 0.1$ Farads.

(a) Write down the general solution to the homogeneous equation *i.e.* when the external voltage $E(t) = 0$. Sketch a typical solution. 8%

(b) By first finding a particular solution, find the general solution to the differential equation when the external voltage $E(t) = 39\cos t$. 7 %

(c) Solve the equation in (b) when the initial charge on the capacitor is $q(0) = 4$ Farad and the initial current is $q'(0) = 0$ amps. 4%

4 (a) Find the Taylor Series, up to and including quadratic terms, of $z = f(x, y) = \ln(x^2 + 2y^2)$ about the point $(1, 1)$. 10%

(b) The least squares line approximation to the points $(0, 14)$, $(1, a)$, $(3, b)$, $(4, 2)$, $(6, 0)$ is $y = 16 - 3x$. Find a and b. 9%

5 (a) Evaluate the matrix product

$$\begin{pmatrix} 2 & 0 \\ 3 & 1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 & -2 \\ 3 & 0 & 2 & 0 \end{pmatrix}.$$

4%

(b) Evaluate the determinant

$$\begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \\ 6 & 1 & 2 \end{vmatrix}.$$

5%

(c) Find the inverse of the 3×3 matrix 10%

$$\begin{pmatrix} 1 & 3 & -1 \\ 0 & -2 & 3 \\ 2 & 1 & 5 \end{pmatrix}.$$