UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2003

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: 2 hours

LECTURER: Dr. E. Gath

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**

Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.

Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.

You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A bungee jumper has acceleration $a = \dfrac{d^2s}{dt^2} = 10\cos(0.5t)$ metres/second^2 at time t. She jumps at time $t = 0$. How far does she travel in the first 3 seconds? 4%

(b) Give the numerical output that results from implementing the Maple command:

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with(student):
evalf(rightsum(exp(x),x=1..3, 2));
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4%

(c) Prove that $I_n \equiv \displaystyle\int_0^{\frac{\pi}{2}} (\cos x)^{2n+1}\,dx$, $n = 0, 1, 2, 3, \ldots$, satisfies the iterative reduction equation $I_n = \left(\dfrac{2n}{2n+1}\right) I_{n-1}$ for $n \geq 1$.

(Hint: integrate by parts with $u = \cos^{2n} x$ and $dv = \cos x\,dx$ and then use a well-known trigonometric identity.) 4%

(d) Find the volume of the solid of revolution that results from revolving the bounded region enclosed between the curve $y = 4 - x^2$ and the x-axis, about the x-axis. 4%

(e) Find all *first and second partial derivatives* of $f(x, y) = \sin(xy)$. 4%

(f) Using the *Improved Euler method*, with step size $h = 0.1$, write down an iterative scheme which approximates the solution of the initial value problem $y' = \cos(x + 2y)$, $\quad y(0) = 2$. 4%

(g) Find the general solution of the differential equation $\dfrac{dy}{dx} - \dfrac{3y}{x} = x^2$. 4%

(h) Evaluate the determinant $\begin{vmatrix} 4 & 1 & 2 \\ -1 & 3 & 4 \\ 1 & 3 & -1 \end{vmatrix}$. 4%

2 Answer part (a) and *any two* of parts (b), (c), (d):

(a) A solid of revolution is generated by revolving about the y-axis the area bounded between $y = e^{-x^2}$ and the x-axis for $0 \leq x \leq 1$. Find the volume of the solid so obtained. (Hint: use cylindrical shells.) 5%

(b) Find the area of the region between $y = \ln x$ and the x-axis, for $1 \leq x \leq e^2$. 7%

(c) Find the arc-length along the curve $y = \dfrac{x^3}{3} + \dfrac{1}{4x}$ for $1 \leq x \leq 3$. 7%

(d) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \dfrac{1}{\sqrt{16 - x^2}}$, for $0 \leq x \leq 2$. 7%

3 The charge $q(t)$, at time t, in the capacitor in the LRC circuit depicted below satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t).$$

Consider a circuit with an inductor with $L = 1$ Henry, a resistor with $R = 4$ ohms and a capacitor with $C = 0.25$ Farads.

(a) Write down the general solution to the homogeneous equation *i.e.* when the external voltage $E(t) = 0$. Sketch a typical solution. 8%

(b) By first finding a particular solution, find the general solution to the differential equation when the external voltage $E(t) = \cos 2t + \sin 2t$. 7 %

(c) Solve the equation in (b) when the initial charge on the capacitor is $q(0) = 1$ Farad and the initial current is $q'(0) = 0$ amps. 4%

4 (a) Find the Taylor Series, up to and including quadratic terms, of $z = f(x,y) = \ln(x^2 + 3y)$ about the point $(1,1)$. 10%

(b) The least squares line approximation to the points $(-2,9)$, $(-1,6)$, $(0,1)$, $(1,a)$, $(2,b)$ is $y = 2 - \dfrac{17x}{5}$. Find a and b. 9%

7+2%

5 (a) Find all solutions of each system of linear equations:

(i)
$$\begin{array}{rrrcl} 2x & +y & -3z & = & 1 \\ x & -2y & +z & = & 4 \\ x & -y & & = & 1 \end{array}$$

(ii)
$$\begin{array}{rrrcl} x & & +z & = & 3 \\ 2x & +y & +3z & = & 4 \\ x & -y & & = & 5 \end{array}$$

5%+5%

(b) Find the inverse of the 3×3 matrix 9%

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & -3 \\ 3 & 0 & 2 \end{bmatrix}.$$