UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2007

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A particle, shot with an initial velocity of 1000 m/s, is slowed due to air resistance that results in a negative acceleration of $-3000e^{-2t}$ m/s^2. How long will it take until the particle stops? How far does the particle move before it stops? 4%

(b) Find the volume of the solid obtained by rotating the plane region bounded by the curve $y = \sin x$ and the x-axis for $0 \le x \le \pi$, about the x-axis. 4%

(c) Obtain an iterative reduction formula for $I_n = \displaystyle\int_1^e (\ln x)^n \, dx$. (Hint: integrate by parts.) Evaluate I_0. Then, using the reduction formula obtained, evaluate I_1, I_2, and I_3. 4%

(d) Find all *first and second partial derivatives* of $f(x, y) = y\,\sin(xy)$. 4%

(e) Write down the iterative scheme of the *Improved Euler method* applied to the initial value problem $y' = \cos(x + y)$, $y(0) = 0$ with step size $h = 0.1$. Evaluate the approximations of $y(0.1)$ and $y(0.2)$ obtained using this scheme. 4%

(f) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{2x + \cos x}{3y^2}$, with $y(0) = 1$. 4%

(g) Evaluate the determinant $\begin{vmatrix} 3 & 1 & -1 \\ 3 & 0 & 4 \\ 6 & -2 & 5 \end{vmatrix}$. 4%

(h) Prove that there exist 2×2 *non-zero* matrices A and B such that

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(give an example). 4%

2 (a) A solid of revolution is obtained by rotating about the y-axis the area bounded between $y = \frac{1}{x+1}$ and $y = \frac{1}{x+2}$ for $0 \le x \le 1$. Find the volume of the solid obtained. (Hint: use cylindrical shells.) 6%

(b) Find the arc-length along the curve $y = \left(4 - x^{2/3}\right)^{3/2}$ for $1 \le x \le 8$. 6%

(c) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \ln x$ for $1 \le x \le e$. 7%

3 (a) Find general solutions of the given differential equations: 4%+4%
(i) $y'' - 7y' + 12y = 0$, (ii) $y'' - 4y' + 5y = 0$.

(b) Find a particular solution to the given differential equation: 5%+3%
$y'' - 7y' + 12y = 170 \sin x$.
Then find the general solution of this equation.

(c) Solve the equation in (b) when $y(0) = 2$, $y'(0) = 1$. 3%

4 (a) Find the Taylor Series, up to and including quadratic terms, 10%
of $z = f(x, y) = (x^2 + y) \ln(xy)$ about the point $(1, 1)$.

(b) Find the least squares line approximation to the points: 7%+2%
$(0, 2)$, $(1, 0)$, $(2, 3)$, $(3, 3)$, $(4, 5)$, and $(5, 6)$.
Sketch the points and the least squares line on the one graph.

5 (a) Find all solutions of each system of linear equations: 4%+4%+2%

(i) $$\begin{aligned} 3x + 2y + 5z &= 33 \\ 6x + 5y + z &= 55 \\ -3x - 6y + 30z &= 9 \end{aligned}$$;

(ii) $$\begin{aligned} 3x + 2y + 5z &= 33 \\ 6x + 5y + z &= 55 \end{aligned}$$;

(iii) $3x + 2y + 5z = 33$.

(b) Find the inverse of the matrix 9%

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & -3 & 1 \\ -8 & 7 & 4 \end{bmatrix}.$$