UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering

Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE TITLE: Engineering Mathematics 2

MODULE CODE: MA4002

SEMESTER: Spring 2019

LECTURER/EXAMINER: Prof. N. Kopteva

EXTERNAL EXAMINER: Prof. R. E. Wilson

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

PERCENTAGE OF TOTAL MARKS: 30% (continuous assessment) +70% (final exam)

INSTRUCTIONS TO CANDIDATES:

- **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
- **Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.**
- **Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted.**

1. (a) An object has acceleration $a(t) = 2(t+1)^{-1/3}$ $\mathrm{m\,s^{-2}}$ at time t. The initial velocity at time $t = 0$ is $v = 1$ metres/second. How far does it travel in the first 7 seconds? 3%

   (b) Consider the plane region bounded by the curve $y = x^3$ and the line $y = 1$ for $0 \le x \le 1$. Sketch this region. Find the volume of each of the <u>two solids</u> obtained by rotating this plane region (i) about the x-axis; (ii) about the y-axis. 4%

   (c) Obtain an iterative reduction formula for $I_n = \displaystyle\int_0^3 e^{x/3}\, x^n\, dx$. Evaluate I_0. Then, using the reduction formula obtained, evaluate I_1 and I_2. 5%

   (d) Find all *first and second partial derivatives* of $f(x, y) = e^{xy-2}$. 5%

   (e) Find the linearization of $f(x, y) = e^{xy-2}$ about the point $(2, 1)$. (You may use the results of part (d).) 4%

   (f) Solve the differential equation $\dfrac{dy}{dx} + \dfrac{3}{x}y = \dfrac{2}{x^2}$ (for $x > 1$), subject to the initial condition $y(1) = 5$. 4%

   (g) Evaluate the determinants $\begin{vmatrix} 3 & 1 & -5 \\ 2 & 0 & -6 \\ 7 & 1 & 0 \end{vmatrix}$ and $\begin{vmatrix} 3 & 1 & 0 & -5 \\ 5 & 1 & 2 & 8 \\ 2 & 0 & 0 & -6 \\ 7 & 1 & 0 & 0 \end{vmatrix}$. 5%

   (h) Prove that there exist 3×3 non-zero matrices A and B such that
   $$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ (give an example).}$$ 4%

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2. (a) A solid of revolution is obtained by rotating about the y-axis the area bounded between $y = \dfrac{x+1}{(x+2)(x+3)(x+4)}$ and the x-axis for $0 \le x \le 2$. Find the volume of the solid obtained. 6%

   (b) Find the arc length of the curve $y = (2x+2)^{3/2}$ for $0 \le x \le 4$. 6%

   (c) Find the mass and the centre of mass of a rod with mass density $\rho(x) = x \sin x$ for $0 \le x \le \pi$. 6%

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3. (a) Find general solutions of the given differential equations: 2%+2%
(i) $y'' - 4y' + 3y = 0$, (ii) $y'' - 4y' + 5y = 0$.

(b) Find a particular solution for each of the given differential equations: 6%+6%+2%
(i) $y'' - 4y' + 3y = 2e^x + 5\sin x$,
(ii) $y'' - 4y' + 5y = 2e^x + 5\sin x$.
Then find the general solutions of these equations.
(You may use the results of part (a).)

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4. (a) Find the Taylor Series, up to and including quadratic terms, 9%
of $z = f(x, y) = \dfrac{1}{x + y^2}$ about the point $(0, 1)$.

(b) It is known that the quantities $z > 0$ and $t > 0$ are related by the formula $z = \alpha t^{\beta}$, with some unknown constants $\alpha > 0$ and β. By writing this as $\ln z = \beta \ln t + \ln \alpha$, one can use the method of <u>*least squares*</u> to find the best-fit line relating $\ln z$ to $\ln t$ and hence find an approximation of the constants α and β. For the given data points

$(t,\ z) = (1,\ 4),\ (2,\ 6),\ (3,\ 7),\ (4,\ 8),\ (5,\ 8.5)$,

use this method to find an approximation of the constants α and β. 9%

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5. (a) Find all solutions of each system of linear equations: 4%+4%

$$\text{(i)}\quad \begin{aligned} x - 2y + 3z &= 2 \\ 3x - 5y + 8z &= -3 \\ -2x + 5y - 7z &= -6 \\ x - 3y + z &= -4 \end{aligned} \quad ; \quad \text{(ii)}\quad \begin{aligned} x - 2y + 3z &= 2 \\ 3x - 5y + 8z &= -3 \\ -2x + 5y - 7z &= -13 \\ x - 3y + z &= -4 \end{aligned}$$

(b) Find the inverse of the matrix 10%

$$\begin{bmatrix} 2 & -1 & 3 & 0 \\ 4 & -1 & 7 & 1 \\ -2 & 6 & 3 & 2 \\ 2 & 1 & 8 & -6 \end{bmatrix}.$$

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END OF PAPER