UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2022

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Prof. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. R.E. Wilson

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) An object has acceleration $a(t) = \dfrac{2}{(t+1)^{3/5}}$ metres/second^2 at time t. The initial velocity at time $t = 0$ is $v = 2$ metres/second. How far does it travel in the first 6 seconds? 3%

(b) Consider the plane region bounded by the curves $y = 3x - 2x^2$ and $y = x^3$ for $x \geq 0$. Find the volume of each of the two solids obtained by rotating this plane region **(i)** about the x-axis; **(ii)** about the y-axis. 5%

(c) Obtain an iterative reduction formula for $I_n = \int_0^1 x^n e^{-x/3} dx$. Evaluate I_0. Then, using the reduction formula obtained, evaluate I_1 and I_2. 5%

(d) Find all *first and second partial derivatives* of $f(x, y) = \sin(x^2 - y^3)$. 4%

(e) Find the linearization of the function $f(x, y) = \sin(x^2 - y^3)$ about the point (2,1). (You may use the results of part (d).) 2%

(f) Solve the differential equation $x\dfrac{dy}{dx} + 5y = \dfrac{4\sin(2x)}{x^3}$ (for $x > 1$), subject to the initial condition $y\left(\dfrac{\pi}{4}\right) = 2$. 4%

(g) Evaluate the three determinants

$$\begin{vmatrix} 2 & 3 & 0 \\ 1 & 4 & -2 \\ -1 & 5 & -3 \end{vmatrix}, \quad \begin{vmatrix} 3 & 0 & 1 \\ 4 & -2 & 4 \\ 5 & -3 & 2 \end{vmatrix}, \text{ and } \begin{vmatrix} 1 & 0 & 0 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 4 & -2 & 4 \\ -1 & 5 & -3 & 2 \end{vmatrix}.$$ 7%

(h) Prove that $\int \dfrac{dx}{x} = \ln|x| + C$ (for $x \neq 0$) from the definition of the indefinite integral. (Hint: consider the cases of $x > 0$ and $x < 0$.) 4%

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2 (a) A solid of revolution is obtained by rotating about the y-axis the area bounded between $y = \dfrac{1}{(x+1)(x+2)^2}$ and the x-axis for $0 \leq x \leq 3$. Find the volume of the solid obtained. 6%

(b) Find the arc-length of the curve $y = x^{3/2}$ for $0 \leq x \leq 1$. 4%

(c) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \ln(x+1)$ for $0 \leq x \leq 4$.
(Hint: use the substitution $u = x + 1$.) 8%

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3 (a) Find general solutions of the given differential equations: 2%+2%

(i) $y'' - 8y' - 9y = 0,$ (ii) $y'' - 2y' + 26y = 0.$

(b) Find a particular solution for each of the given differential equations: 6%+6%+2%

(i) $y'' - 8y' - 9y = 2e^{-x} - 9x,$

(ii) $y'' - 2y' + 26y = 2e^{-x} - 9x.$

Then find the general solutions of these equations.

(You may use the results of part (a).)

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4 (a) Find the Taylor Series, up to and including quadratic terms, of $z = f(x, y) = \ln(xy^3 - 1)$ about the point $(3, 1)$. 9%

(b) It is known that the quantities $z > 0$ and $t > 0$ are related by the formula $z^\alpha = 2t^\beta$, with some unknown constants $\alpha \neq 0$ and β. By writing this as $\alpha \ln z = \beta \ln t + \ln 2$, and then as $\ln z = \dfrac{\beta}{\alpha} \ln t + \dfrac{\ln 2}{\alpha}$, one can use the method of *least squares* to find the best-fit line relating $\ln z$ to $\ln t$ and hence find an approximation of the constants α and β.
For the given data points

$$(t,\, z) = (1,\, 9),\, (3,\, 2),\, (5,\, 7),\, (7,\, 5),\, (9,\, 3),$$

use this method to find an approximation of the constants α and β. 9%

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5 (a) Find all solutions of each system of linear equations: 4%+4%

$$\text{(i)}\quad \begin{aligned} 2x - y + 2z &= 3 \\ 4x - y + 3z &= 4 \\ -2x - y + z &= 9 \\ 6x + y + 3z &= 9 \end{aligned} \quad ; \quad \text{(ii)}\quad \begin{aligned} 2x - y + 2z &= 6 \\ 4x - y + 3z &= 8 \\ -2x - y &= 2 \\ 6x + y + 2z &= 2 \end{aligned}$$

(b) Find the inverse of the matrix 10%

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -1 & 2 & -4 \\ 3 & -1 & -9 & 4 \\ 4 & 3 & 3 & 17 \end{bmatrix}.$$

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