

1 (a) Evaluate the indefinite integral $\int \frac{11\sqrt{x} - 13x}{\sqrt{x}} dx$. 2%

$$-9x^{5/3} + 12x^{11/2} + C$$

1/1, 1/1.

(b) Calculate the area between $y = 5^x + \sin(x^2) + x^{11}$ and the x -axis for $-2 \leq x \leq 2$. 2%

$$A = \int_{-2}^2 (5^x + \sin x^2 + x^{11}) dx = \frac{1}{2} \cdot 1 + 0 + 0 = \frac{1}{2}$$

1/2%, 1/2%, 1/2%

(c) Express as a definite integral and then evaluate the limit of the Riemann sum $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{\sqrt{c_i+1}} \Delta x_i$, where $c_i \in [x_{i-1}, x_i]$, and we use the partition P with $x_i = -1 + \frac{4i}{n}$ for $i = 0, 1, \dots, n$ and $\Delta x_i \equiv x_i - x_{i-1}$. 1%

$$= \int_{-1}^3 \frac{dx}{\sqrt{x+1}} = \left[2\sqrt{x+1} \right]_{-1}^3 = 4$$

1/1

(d) Evaluate $\frac{d}{dx} \int_{x^2-x}^x (\sin t + 1) dt$. 1%

$$0 - (3x^2 - 1) \cdot [8 \sin(x^3 - x) + 1]$$

1/1, 1/1.

(e) Find an upper bound for the error E_S in the Simpson's Rule approximation of the definite integral $\int_0^1 e^{-\sqrt{3}x} dx$, using N subintervals. Choose N such that $E_S \leq 5 \cdot 10^{-6}$. Hint: evaluate $M_4 = \max_{x \in [0,1]} \left| \frac{d^4}{dx^4} e^{-\sqrt{3}x} \right|$. 2%

$$M_4 = (\sqrt{3})^4 = 9$$

$$E_S \leq \frac{(b-a)^5 \cdot M_4}{180 \cdot N^4} = \frac{1 \cdot 9}{180 \cdot N^4} = \frac{1}{20N^4}$$

$$\frac{1}{20 \cdot N^4} \leq 5 \cdot 10^{-6} \Rightarrow N \geq 10$$

1/2%, 1/2%, 1/2%

2. Evaluate the indefinite integral $\int \cos^3 x \cdot \sin^2 x dx$. 3%

$$= \int (1 - \sin^2 x) \cdot \sin^2 x \cdot \cos x dx$$

$$= \int (1 - u^2) u^2 du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

1/1, 1/1, 1/1, 1/1, 1/1