Question 1(a)

$$\int \frac{x^2 - x}{\sqrt{x-1}}\,dx$$

$$u = x-1 \implies I = \int \frac{(u^2+2u+1)-(u+1)}{\sqrt{u}}\,du$$

$$= \int \left(u^{3/2} + u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$$

Question 1(b)

$$A = \int_0^{\pi} \left(2^x + \sin x\right) dx$$

$$= \left(\frac{2^x}{\ln 2} - \cos x\right)\Bigg|_0^{\pi}$$

$$= \boxed{\frac{2^{\pi}-1}{\ln 2} + 2}$$

Question 1(c)

$$a = x_0 = -2, \quad b = x_n = -2+5 = 3$$

$$\implies \int_{-2}^{3} \frac{dx}{(x+3)^2} = -\frac{1}{x+3}\Bigg|_{-2}^{3}$$

$$= -\frac{1}{6} + 1 = \boxed{\frac{5}{6}}$$

Question 1(d)

$$= \left[\sin(2x+1) + (2x+1)^2\right] \cdot \underbrace{(2x+1)'}_{2}$$
$$- \left[\sin(\sqrt{x}) + (\sqrt{x})^2\right] \cdot \underbrace{(\sqrt{x})'}_{\frac{1}{2\sqrt{x}}}$$
$$= \boxed{2\sin(2x+1) + (8x^2+8x+2) - \frac{1}{2\sqrt{x}}\sin(\sqrt{x}) - \frac{\sqrt{x}}{2}}$$
or similar

. . . .

Question 1(e)

$$(x \cdot \cos x)'' = (\cos x - x\sin x)'$$
$$= -2\sin x - x\cos x$$
$$M_2 \le \max_{x\in[0,2]} \left\{ 2\underbrace{|\sin x|}_{\le 1} + \underbrace{|x|}_{\le 2} \cdot \underbrace{|\cos x|}_{\le 1} \right\}$$
use $\boxed{M_2 = 4}$
$$E_T \le \frac{1}{12}\frac{(b-a)^3}{n^2} M_2 = \frac{2^3}{12n^2}4 = \boxed{\frac{8}{3n^2} \le \frac{2}{3}\cdot 10^{-4}}$$
$$\Longrightarrow \boxed{n \ge 200}$$

. . . .

Question 2

$$\int \underbrace{\cos^2(x)}_{(1-\sin^2 x)} \cdot \cos(x) \cdot dx$$
$$\boxed{u = \sin x}$$
$$du = \cos x \cdot dx$$
$$I = \int (1-u^2)\, du \quad 2$$
$$= u - \frac{u^3}{3} + C = \boxed{\sin x - \frac{\sin^3 x}{3} + C}$$

Question 3

$$\bar{f} = \frac{1}{3}\int_1^4 \frac{4x+4}{x^2+4x}$$

Solution I

$$x^2+4x = (x+2)^2 - 4$$

$$u = x+2$$

$$\bar{f} = \frac{1}{3}\int_{u=3}^{u=6} \frac{4u-4}{u^2-4}$$

$$= \frac{1}{3}\int_3^6 \left(\frac{4u}{u^2-4}du - \frac{4}{u^2-4}\right)du$$

$$= \frac{1}{3}\left(2\ln|u^2-4| - \ln\left|\frac{u-2}{u+2}\right|\right)\Big|_3^6$$

Solution II (partial fractions)

$$\frac{4x+4}{x^2+4x} = \frac{A}{x} + \frac{B}{x+4}$$

$$= \frac{1}{x} + \frac{3}{x+4}$$

$$\Longrightarrow \bar{f} = \frac{1}{3}\left(\ln|x| + 3\ln(x+4)\right)\Big|_1^4$$

$$= \boxed{\frac{11}{3}\ln 2 - \ln 5}$$

Question 4

$$\int (8x^3-1)\underbrace{\ln x}_{u}\,dx$$

$$= (2x^4-x)\ln x - \underbrace{\int \frac{2x^4-x}{x}dx}_{\parallel}$$

$$\int (2x^3-1)\,dx$$

$$= \frac{x^4}{2} - x$$

$$= \boxed{(2x^4-x)\ln x - \frac{x^4}{2} + x}$$

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Question 5

$$\frac{x^2+3}{(x+1)^2(x^2+4)} = \boxed{\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+4}}$$

or

$$x^2+3 = A(x+1)(x^2+4) + B(x^2+4) + (Cx+D)(x+1)^2$$

(i) set $x=-1$:

$$4 = 5B \implies \boxed{B = \frac{4}{5}}$$

(ii)
$$x^2+3 = A(x^3+x^2+4x+4) + B(x^2+4) + C(x^3+2x^2+x) + D(x^2+2x+1)$$

$$= x^3 \underset{=0}{\boxed{A+C}} + x^2 \underset{=1}{\boxed{A+B+2C+D}}$$

$$+ x \underset{=0}{\boxed{4A+C+2D}} + \underset{=3}{\boxed{4A+4B+D}}$$

$$A+C=0 \implies \underline{C=-A}$$

$$1 = A + \underbrace{B}_{=\frac{4}{5}} + \underbrace{2C}_{-2A} + D = \frac{4}{5} - A + D \implies \underline{D = A + \frac{1}{5}}$$

$$0 = 4A + \underbrace{C}_{=-A} + \underbrace{2D}_{=2A+\frac{2}{5}} = 5A + \frac{2}{5} \implies \boxed{A = -\frac{2}{25}}$$

$$\Downarrow$$

$$\boxed{C = \frac{2}{25}}$$

$$\boxed{D = \frac{3}{25}}$$

$$I = \int \left(-\frac{2}{25}\cdot\frac{1}{x+1} + \frac{4}{5}\frac{1}{(x+1)^2} + \frac{2}{25}\frac{x}{x^2+4} + \frac{3}{25}\frac{1}{x^2+4}\right)dx$$

$$= \boxed{-\frac{2}{25}\ln|x+1| - \frac{4}{5}\frac{1}{x+1} + \frac{1}{25}\ln(x^2+4) + \frac{3}{50}\tan^{-1}\left(\frac{x}{2}\right) + C}$$