Question 1(a)

$$\int \frac{x-2}{\sqrt[3]{x+4}}\,dx = \int \frac{u-6}{u^{1/3}}\,du \quad \Big\} 1\%$$

$u = x+4$

$$= \int u^{2/3}\,du - 6\int u^{-1/3}\,du$$

$$= \boxed{\frac{3}{5}(x+4)^{5/3} - \frac{9}{6\cdot\frac{3}{2}}(x+4)^{2/3} + C} \quad \Big\} \text{each } \int = \tfrac{1}{2}\%$$

Question 1(b)

$$A = \int_1^3 \left(2^x - x^{-2}\right)dx \quad \Big\} 0.5\%$$

$$= \frac{2^x}{\ln 2}\Big|_1^3 + \frac{1}{x}\Big|_1^3 \quad \Big\} 0.5\% \text{ each term}$$

$$= \boxed{\frac{6}{\ln 2} - \frac{2}{3}} \quad \Big\} 0.5\%$$

Question 1(c)

$x_0 = -1, \quad x_n = 1$

$$\Rightarrow \underbrace{\int_{-1}^{1} \frac{dx}{x^2+1}}_{0.5\% \text{ for limits; } 0.5\% \text{ for the function } \frac{1}{x^2+1}} = \underbrace{\tan^{-1}x\Big|_{-1}^{1}}_{0.5\%} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \underbrace{\frac{\pi}{2}}_{0.5\%}$$

Question 1(d)

$$\sin\sqrt{4+x^3}\cdot(3+x^3)' - \sin\sqrt{1+x^2}\cdot(x^2)'$$

$$= \underbrace{3x^2\sin\sqrt{4+x^3}}_{0.5\%} - \underbrace{2x\sin\sqrt{1+x^2}}_{0.5\%}$$

Question 1(e)

$$E = \frac{(3-0)^3}{12n^2}\cdot M_2 \quad \left.\begin{array}{l}\\ \\ \end{array}\right\} \text{0.5\% for the formula}$$ 0.5%

$$(e^{-2x})'' = 4e^{-2x} \Longrightarrow \boxed{M_2 = \max_{x\in[0,3]} 4|e^{-2x}| = 4}$$

$$\Longrightarrow \boxed{E = \frac{3^3}{12n^2}\cdot 4 = \frac{9}{n^2}} \quad 0.5\%$$

$$\frac{9}{n^2} \le 10^{-4} \Longrightarrow \boxed{n \ge 300} \quad 0.5\%$$

Question 2 (Lecture 6, p. 11)

$$\int \sin^4 x\,dx = \int\left(\frac{1-\cos 2x}{2}\right)^2 dx \quad \left.\begin{array}{l}\\ \\ \end{array}\right\} 1\%$$

$$= \frac{1}{4}\int\left(1 - 2\cos(2x) + \cos^2(2x)\right)dx$$

$$= \underbrace{\frac{x}{4}}_{0.5\%} - \underbrace{\frac{1}{4}\sin(2x)}_{0.5\%} + \underbrace{\frac{1}{8}\left(x + \frac{1}{4}\sin(4x)\right)}_{1\%} + C$$

(i) from log tables

(ii) using $\cos^2 2x = \frac{1+\cos(4x)}{2}$

$$= \boxed{\frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$$

Question 3

$$\bar{f} = \frac{1}{2}\int_{-2}^{0} \frac{x+3}{x^2+4x+5}\,dx \quad \Big\} 0.5\%$$

$$x^2+4x+5 = (x+2)^2+1 \Longrightarrow u = x+2 \Big\} 1\%$$

$$\bar{f} = \frac{1}{2}\int_{u=0}^{u=2} \frac{u+1}{u^2+1}\,du$$

$$= \frac{1}{2}\int_{0}^{2} \frac{u}{u^2+1}\,du + \frac{1}{2}\int_{u=0}^{u=2} \frac{du}{u^2+1}$$

$$= \frac{1}{4}\ln(u^2+1)\Big|_0^2 + \frac{1}{2}\tan^{-1}u\Big|_0^2 \quad \Big\} \text{0.5\% each term}$$

$$= \frac{1}{4}\ln 5 + \frac{1}{2}\tan^{-1}2 \quad \Big\} 0.5\%$$

Question 4

$$\int \underbrace{x^2}_{u}\,\underbrace{e^{3x}\,dx}_{dv} \quad \Big\} \text{1.5\% (including 0.5\% for choosing } u \text{ \& } dv)$$

$$= \frac{e^{3x}}{3}x^2 - \frac{2}{3}\int \underbrace{x}_{u}\,\underbrace{e^{3x}\,dx}_{dv}$$

$$= \left(x\frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx\right) \Big\} \text{1.5\% (as above)}$$

$$= \left(\frac{x}{3} - \frac{1}{9}\right)e^{3x} \quad \Big\} 0.5\%$$

$$= \boxed{e^{3x}\left(\frac{x^2}{3} - \frac{2}{9}x + \frac{2}{27}\right) + C} \quad \Big\} 0.5\%$$

Question 5

$$\frac{9-x}{x(x-3)^2} = \boxed{\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}}$$

2% (partial marks possible)

$$9 - x = A(x-3)^2 + Bx(x-3) + Cx$$

$x = 0$: $9 = 9A \Rightarrow \boxed{A = 1}$

$x = 3$: $6 = 3C \Rightarrow \boxed{C = 2}$

$x = 2$: $7 = \underbrace{A \cdot 1}_{1} + \underbrace{B \cdot 2(-1)}_{-2B} + \underbrace{C \cdot 2}_{4}$

$2B = -2$, $\boxed{B = -1}$

0.5% for each

$$\int \left(\frac{1}{x} - \frac{1}{x-3} + \frac{2}{(x-3)^2} \right) dx$$

$$= \underbrace{\boxed{\ln|x| - \ln|x-3| - \frac{2}{x-3} + C}}$$

0.5% for each term