Question 1(a)

$$\int \frac{x^2}{\sqrt{x^3+1}}\,dx = \int \frac{du/3}{\sqrt{u}} =$$

$u = x^3+1$ +0.5%

$du = 3x^2 \cdot dx \Rightarrow x^2\,dx = \frac{du}{3}$ } +0.5%

$$= \underbrace{\frac{1}{3}\int u^{-1/2}\,du}_{+0.5\%} = \frac{1}{3}\cdot 2\sqrt{u} + C = \boxed{\frac{2}{3}\sqrt{x^3+1} + C}$$ } +0.5%

Question 1(b)

$$A = \int_0^1 \left(3^x - \frac{1}{x+1}\right) dx$$ } 0.5%

$$= \left(\underbrace{\frac{3^x}{\ln 3}}_{0.5\%} - \underbrace{\ln|x+1|}_{0.5\%}\right)\Big|_0^1$$

$$= \frac{3^1 - 3^0}{\ln 3} - (\ln 2 - \underbrace{\ln 1}_{=0}) = \boxed{\frac{2}{\ln 3} - \ln 2}$$ 0.5%

Question 1(c)

$x_i = -1 + \frac{3i}{n} \Rightarrow \underbrace{x_0 = -1 \quad x_n = 2}_{0.5\%}$

$$\Rightarrow \int_{-1}^{2} \sin(2x)\,dx$$ } 0.5%

$$= \underbrace{-\frac{\cos(2x)}{2}}_{0.5\%}\Big|_{-1}^{2} = \boxed{-\frac{\cos 4}{2} + \frac{\cos(-2)}{2}}$$ 0.5%

or

$$\boxed{\frac{\cos 2}{2} - \frac{\cos 4}{2}}$$

Question 1(d)

$$\frac{d}{dx}\left(\int_{2x}^{\cos x} \sqrt{\sin t + 1}\, dt\right)$$

$$= \overbrace{\underbrace{-\sin x}_{(\cos x)'} \cdot \sqrt{\sin(\cos x)+1}}^{0.5\%} \overbrace{- \underbrace{2}_{(2x)'} \sqrt{\sin(2x)+1}}^{0.5\%}$$

if "+" $\Rightarrow$ deduct 0.25 %

Question 1(e)

$$E_S \le \frac{(b-a)^5}{180 \cdot N^4} \cdot M_4$$

$$M_4 = \max_{x\in[1,2]} |9 \cos(\sqrt{3}x)| \le 9 \quad \} \; 0.5\%$$

$$E_S \le \frac{(2-1)^5}{180 \cdot N^4} \cdot 9 = \boxed{\frac{1}{20 \cdot N^4}} \quad 0.5\%$$

$$E_S \le 5 \cdot 10^{-10} \Leftarrow \underbrace{\frac{1}{20N^4} \le 5 \cdot 10^{-10}}_{0.5\%}, \text{ or } N^4 \ge \frac{10^{10}}{20 \cdot 5} = 10^8$$

$$\boxed{N \ge 100} \quad 0.5\%$$

Question 2

$$\int \sin^5 x\, dx = \int (\underbrace{\sin^2 x}_{=1-\cos^2 x})^2 \cdot \sin x\, dx \stackrel{(=)}{}$$

$\boxed{u = \cos x}$ 0.5%

$$du = -\sin x \cdot dx \Longrightarrow \sin x \cdot dx = -du$$

$$(=) \int \underbrace{(1-u^2)^2}_{0.5\%} \underbrace{(-du)}_{0.5\%}$$

$$= \underbrace{-\int (1 - 2u^2 + u^4)\, du}_{0.5\%} = \overbrace{-u + \frac{2u^3}{3} - \frac{u^5}{5} + C}^{0.5\%}$$

$$= \boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C} \quad 0.5\%$$

Question 3

$$f(x) = \frac{x+6}{x^2+4x+4}$$

$$\bar{f} = \frac{1}{2-(-1)} \int_{-1}^{2} \frac{x+6}{x^2+4x+4}\,dx \quad \Big\} 0.5\%$$

$$= \frac{1}{3}\int_{-1}^{2} \frac{x+6}{(x+2)^2}\,dx = \frac{1}{3}\int_{u=1}^{u=4} \frac{u+4}{u^2}\,du \Big\} 0.5\%$$

$u = x+2$ $+0.5\%$

+0.5% each term

$$= \underbrace{\frac{1}{3}\int_1^4 \frac{du}{u} + \frac{4}{3}\int_1^4 \frac{du}{u^2}}_{+0.5\%} = \left(\frac{1}{3}\ln|u| - \frac{4}{3}u^{-1}\right)\Big|_1^4$$

$$= \frac{1}{3}\ln 4 - \frac{4}{3}\left(\frac{1}{4}-1\right) = \underbrace{\boxed{\frac{1}{3}\ln 4 + 1}}_{+0.5\% \text{ each term}}$$

Question 4

$$I = \int \underbrace{e^x}_{u} \underbrace{\sin\left(\frac{x}{2}\right)dx}_{dv} = e^x\left(-2\cos\left(\frac{x}{2}\right)\right) - \int\left(-2\cos\left(\frac{x}{2}\right)\right)e^x\,dx$$

$$\underbrace{du = e^x dx \qquad v = -2\cos\left(\frac{x}{2}\right)}_{0.5\%}$$

$$= -2e^x\cos\left(\frac{x}{2}\right) + 2\int \underbrace{e^x}_{u}\underbrace{\cos\left(\frac{x}{2}\right)dx}_{dv} \quad \Big\} +1\%$$

$$\underbrace{du = e^x dx \qquad v = 2\sin\left(\frac{x}{2}\right)}_{+0.5\%}$$

$$= -2e^x\cos\left(\frac{x}{2}\right)$$

$$+2\left(e^x \cdot 2\sin\left(\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\cdot e^x\,dx\right) \Big\} +1\%$$

$$= -2e^x\cos\left(\frac{x}{2}\right) + 4e^x\sin\left(\frac{x}{2}\right) - 4I$$

recognise this +0.5%

$$\Rightarrow \boxed{I = -\frac{2}{5}e^x\cos\left(\frac{x}{2}\right) + \frac{4}{5}e^x\sin\left(\frac{x}{2}\right) + C} \quad +0.5\%$$

Question 5

$$f = \frac{4}{(x^2+1)(x-1)(x+1)}$$

$$= \underbrace{\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}}_{0.5\%\ \text{for each of the 4 terms}}$$

$$4 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$$

use $x=1$

$$4 = A\cdot 2\cdot 2 + B\cdot 0\cdot C\cdot 0 \Longrightarrow \boxed{A=1}$$

$x=-1$

$$4 = A\cdot 0 + B(-2)\cdot 2 + C\cdot 0 \Longrightarrow \boxed{B=-1}$$

$$4 = A(x^3+x^2+x+1) + B(x^3-x^2+x-1) + C(x^3-x) + D(x^2-1)$$

$$4 = x^3\underbrace{(A+B+C)}_{=0} + x^2\underbrace{(A-B+D)}_{} + x(A+B-C) + (A-B-D)$$

$$\boxed{C} = -(A+B) \boxed{=0}$$

$$A-B+D=0$$

$$\Longrightarrow \boxed{D = B-A = -2}$$

+0.5% for each of A, B, C, D correct

4

$$f = \frac{1}{x-1} + \frac{-1}{x+1} + \frac{0x+(-2)}{x^2+1}$$

$$\int f(x)\,dx = \boxed{\underbrace{\ln|x-1| - \ln|x+1|}_{+0.5\%} - \underbrace{2\tan^{-1}x}_{0.5\%} + C}$$