Question 1(a)

$$u = x^5+7 \quad \} 0.5\%$$

$$\left.\begin{aligned} du &= (x^5+7)'\,dx \\ &= 5x^4\,dx \\ \tfrac{1}{5}\,du &= x^4\,dx \end{aligned}\right\} 0.5\%$$

$\Sigma = 2\%$

$$I = \int \frac{\frac{1}{5}\,du}{\sqrt{u}} = \underbrace{\frac{2}{5}\sqrt{u}+C}_{0.5\%} = \underbrace{\boxed{\frac{2}{5}\sqrt{x^5+7}+C}}_{0.5\%}$$

. . . .

Question 1(b)

$$A = \int_0^1 \left(2^x - \frac{1}{(x+1)^2}\right)dx \quad \} 0.5\%$$

$$= \overbrace{\frac{2^x}{\ln 2}\Big|_0^1}^{0.5\%} + \overbrace{\frac{1}{x+1}\Big|_0^1}^{0.5\%}$$

$$= \boxed{\frac{1}{\ln 2} - \frac{1}{2}} \quad 0.5\%$$

$\Sigma = 2\%$

. . . .

Question 1(c)

$$x_0 = -2, \quad x_n = 2 \implies (-2, 2) \quad \} 0.5\%$$

$$\lim_{n\to\infty} \sum_{n=1}^{n} \left(x_i^2 + \sin(x_i^3)\right)\Delta x = \underbrace{\int_{-2}^{2} \left(x^2+\sin(x^3)\right)dx}_{0.5\%}$$

$$= \underbrace{\underbrace{\int_{-2}^{2} x^2\,dx}_{\frac{x^3}{3}\Big|_{-2}^{2} = \frac{16}{3}}}_{0.5\%} + \underbrace{\underbrace{\int_{-2}^{2} \underbrace{\sin(x^3)}_{\text{odd function}}\,dx}_{0}}_{0.5\%} = \boxed{\frac{16}{3}}$$

1

$\Sigma = 2\%$

Question 1(d)

$\Sigma = 1\%$

$$= (x+11)' \ln([x+11] + \sin(x+11)) - (x^2+1)' \ln(x^2+1+\sin(x^2+1))$$

$$= \underbrace{\ln(x+11+\sin(x+11))}_{0.5\%} \underbrace{-2x\ln(x^2+1+\sin(x^2+1))}_{0.5\%}$$

Question 1(e)

$$E_S = \frac{1}{180}\frac{(b-a)^5}{N^4} M_4 \quad \Big\} 0.5\%$$

$\Sigma = 2\%$

$$M_4 = \max_{x\in[0,9]} |10^4 e^{-10x}| = 10^4 \quad \Big\} 0.5\%$$

$$E_S = \frac{1}{180}\frac{(9-0)^5}{N^4} 10^4 = \overbrace{\frac{1}{20}\left(\frac{90}{N}\right)^4}^{0.5\%}$$

$$E_S \le 5\cdot 10^{-2} \Longleftarrow \frac{1}{20}\left(\frac{90}{N}\right)^4 \le 5\cdot 10^{-2} \text{ so } \underbrace{\boxed{N \ge 90}}_{0.5\%}$$

Question 2

$$I = \int \overbrace{\tfrac{1}{2}(1-\cos(2x))}^{0.5} \cdot \overbrace{\tfrac{1}{2}(1+\cos 2x)}^{0.5} \, dx$$

$$= \overbrace{\frac{1}{4}\int \Big(1 - \underbrace{\cos^2(2x)}_{\text{II}}\Big)\, dx}^{0.5} = \underbrace{\frac{1}{4}\int\left(\frac{1}{2} - \frac{1}{2}\cos(4x)\right)dx}_{0.5}$$

$$\tfrac{1}{2}(1+\cos(4x)) \leftarrow 0.5\%$$

$$= \frac{1}{8}\left(x - \frac{\sin(4x)}{4}\right) + C = \underbrace{\boxed{\frac{x}{8} - \frac{\sin(4x)}{32} + C}}_{0.5\%}$$

Note: one can alternatively use $\sin x \cdot \cos x = \frac{1}{2}\sin 2x$

$\Sigma = 3\%$

$\Sigma = 4\%$

Question 3

$$\bar{f} = \frac{1}{4-2}\int_2^4 \frac{x-5}{x^2+4x+3}\,dx \quad \Big\}\ 0.5$$

$$x^2+4x+3 = (x+2)^2-1 = u^2-1 \quad \text{with } \boxed{u = x+2} \Big\}\ 0.5$$

$$\bar{f} = \frac{1}{2}\int_{x=2}^{x=4} \frac{(u-2)-5}{u^2-1}\,du = \frac{1}{2}\int_{u=4}^{u=6}\left(\frac{u}{u^2-1} - \frac{7}{u^2-1}\right)du \Big\}\ 0.5 +$$

0.5 for limits $u = 4; 6$

$$= \underbrace{\frac{1}{4}\ln|u^2-1|}_{0.5}\Big|_4^6 - \underbrace{\frac{7}{4}\ln\left|\frac{u-1}{u+1}\right|}_{0.5}\Big|_4^6$$

$$= \boxed{\underbrace{\frac{1}{4}(\ln 35 - \ln 15)}_{0.5} - \underbrace{\frac{7}{4}\left(\ln\frac{5}{7} - \ln\frac{3}{5}\right)}_{0.5}}$$

$$= \boxed{2\ln 7 - \frac{7}{2}\ln 5 + \frac{3}{2}\ln 3}$$

any of these two answers is correct

Note: partial fraction representation may be used as well

Question 4

$$I = \int x(\ln x)^2\,dx$$

0.5% — $u = (\ln x)^2$, $dv = x\,dx$ $\Big\} \Longrightarrow \Big\{$ $du = 2\ln x\cdot\frac{1}{x}dx$, $v = \frac{x^2}{2}$

$$I = (\ln x)^2\cdot\frac{x^2}{2} - \int\frac{x^2}{2}\cdot 2\ln x\cdot\frac{1}{x}\,dx \Big\}\ 1\%$$

$$= \frac{x^2}{2}\ln^2 x - \int x\ln x\,dx$$

0.5% — $u = \ln x$, $dv = x\,dx$ $\Big\} \Longrightarrow \Big\{$ $du = \frac{1}{x}dx$, $v = \frac{x^2}{2}$

$$I = \frac{x^2}{2}\ln^2 x - \left(\ln x\cdot\frac{x^2}{2} - \underbrace{\int\frac{x^2}{2}\cdot\frac{1}{x}\,dx}_{\frac{1}{2}\int x\,dx = \frac{x^2}{4}}\right) \Big\}\ 1\%$$

0.5%

$$\boxed{I = \frac{x^2}{2}\ln^2 x - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C} \Big\}\ 0.5\%$$

$\Sigma = 4\%$

Question 5

$$\frac{4}{(x^2-1)(x+1)} = \frac{4}{(x-1)(x+1)^2}$$

$$= \underbrace{\frac{A}{x-1}}_{0.5\%} + \underbrace{\frac{B}{x+1}}_{0.5\%} + \underbrace{\frac{C}{(x+1)^2}}_{1\%}$$

Multiply by $(x-1)(x+1)^2$:

$$4 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

Set $x=-1$: $\quad 4 = A\cdot 0 + B\cdot 0 + C\cdot(-2)$

$$\boxed{C=-2} \quad 0.5$$

$x=1$: $\quad 4 = A\cdot 4 + B\cdot 0 + C\cdot 0$

$$\boxed{A=1} \quad 0.5$$

$x=0$: $\quad 4 = \underbrace{A}_{\substack{\shortparallel\\1}}\cdot 1 + B\cdot(-1) + \underbrace{C}_{\substack{\shortparallel\\-2}}\cdot(-1)$

$$4 = 1 - B + 2$$

$$\boxed{B=-1} \quad 0.5$$

$$\int \underbrace{\left(\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{(x+1)^2}\right)}_{0.5} dx$$

$$= \boxed{\ln|x-1| - \ln|x+1| + \frac{2}{x+1} + C} \quad 1\%$$

(partial marks possible)