2019

MA4002 Engineering Mathematics 2 Prof. N. Kopteva Spring Marks

Question 1(a) 2%

$$I = \int \frac{1}{x\sqrt{\ln x}}\,dx$$

$$\underbrace{u = \ln x}_{0.5\%} \longrightarrow du = \frac{1}{x}\,dx$$

$$\rightarrow I = \underbrace{\int \frac{du}{\sqrt{u}}}_{0.5\%} = \underbrace{2\sqrt{u} + C}_{0.5\%}$$

$$= \boxed{2\sqrt{\ln x} + C} \quad \} \ 0.5\%$$

Question 1(b) 2%

$$A = \int_0^2 \left(5^x - \frac{8}{(x+2)^3}\right) dx \quad \} \ 0.5\%$$

$$= \underbrace{\frac{5^x}{\ln 5}\Big|_0^2}_{0.25\%} - \underbrace{8\,\frac{1}{(x+2)^2\cdot(-2)}\Big|_0^2}_{0.5\%}$$

$$= \underbrace{\frac{24}{\ln 5}}_{0.25\%} + \frac{4}{(x+2)^2}\Big|_0^2 = \frac{24}{\ln 5} + \underbrace{4\left(\frac{1}{16} - \frac{1}{4}\right)}_{=\underbrace{-\frac{3}{4}}_{0.5\%}}$$

$$= \boxed{\frac{24}{\ln 5} - \frac{3}{4}}$$

Question 1(c) 2%

$$x_i = \frac{2i}{n} \Longrightarrow \underbrace{x_0 = 0, \quad x_n = 2}_{0.5\%}$$

$$\Delta x = x_i - x_{i-1} = \frac{2}{n}$$

$$\frac{\Delta x}{2} = \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \exp(-2x_i)\cdot\frac{\Delta x}{2}$$

$$= \underbrace{\frac{1}{2}}_{0.5\%} \underbrace{\int_0^2 \exp(-2x)\,dx}_{0.5\%}$$

$$= \underbrace{\frac{\exp(-2x)}{-4}\Big|_0^2 = \boxed{\frac{1-e^{-4}}{4}}}_{0.5\%}$$

Question 1(d) 1%

$$\frac{d}{dx}\left(\int_{x^3+x}^{x+1} \exp(t\cdot\sin t)\,dt\right)$$

$$= \underbrace{1\cdot\exp\big((x+1)\cdot\sin(x+1)\big)}_{0.5\%} \underbrace{-(3x^2+1)\cdot\exp\big((x^3+x)\cdot\sin(x^3+x)\big)}_{0.5\%}$$

Question 1(e) 2%

$$\underbrace{x^4}_{\boxed{\text{even}}}\ 0.25\%,\quad \underbrace{\sin(x^5+x)}_{\boxed{\text{odd}}}\ 0.5\%,\quad \underbrace{x^4+\sin(x^5+x)}_{\boxed{\text{neither}}}\ 0.25\%$$

$$I = \underbrace{\int_{-1}^{1} x^4\,dx}_{\frac{x^5}{5}\Big|_{-1}^{1} = \frac{2}{5}} + \underbrace{\int_{-1}^{1} \underbrace{\sin(x^5+x)}_{\text{odd}}\cdot dx}_{0} = \boxed{\frac{2}{5}}$$

0.5% 0.5%

Question 2 3%

$$I = \int \sin^2 x\cdot(\cos^2 x)^2\cdot d(\sin x)$$

$$\underbrace{u=\sin x}_{0.5\%} \Longrightarrow I = \int u^2 \underbrace{(1-u^2)^2}_{1-2u^2+u^4}\,du \Big\}\ 1\%$$

$$= \int (u^2 - 2u^4 + u^6)\,du \Big\}\ 0.5\%$$

$$= \frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7} + C \Big\}\ 0.5\%$$

$$= \boxed{\frac{(\sin x)^3}{3} - \frac{2}{5}(\sin x)^5 + \frac{1}{7}(\sin x)^7 + C}\ 0.5\%$$

Question 3

$$\bar{f} = \frac{1}{1}\int_3^4 \frac{x-2}{x^2-6x+10} \quad \Big\}\ 0.5\% \qquad 4\%$$

$$x^2 - 6x + 10 = (x-3)^2 + 1 \ \Big\}\ 0.5\% \qquad \left.\begin{array}{l} x = 3 \to u = 0 \\ x = 4 \to u = 1 \end{array}\right\}\ 0.5\%$$

$$\Rightarrow \quad u = x - 3, \quad du = dx,$$

$$\bar{f} = \underbrace{\int_{u=0}^{u=1} \frac{u+1}{u^2+1}\, du}_{0.5\%} =$$

$$= \int_0^1 \frac{u}{u^2+1}\, du + \int_0^1 \frac{du}{u^2+1}$$

$$= \underbrace{\frac{1}{2}\ln(u^2+1)\Big|_0^1}_{0.5\%} + \underbrace{\tan^{-1}(u)\Big|_0^1}_{0.5\%}$$

$$= \frac{1}{2}\ln 2 + \tan^{-1} 1 = \boxed{\underbrace{\frac{1}{2}\ln 2}_{0.5\%} + \underbrace{\frac{\pi}{4}}_{0.5\%}}$$

Question 4 4%

$$I = \int x^3 \underbrace{(\ln x)^2}_{u} dx$$

$$\left.\begin{array}{l} u \longrightarrow du = \frac{2\ln x}{x} dx \\ dv = x^3 dx \longrightarrow v = \frac{x^4}{4} \end{array}\right\}\ 0.5\%$$

$$I = (\ln x)^2 \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{2\ln x}{x} dx$$

$$= \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\int x^3 \cdot \underbrace{\ln x}_{u} \cdot dx \longrightarrow 1\%$$

$$\left.\begin{array}{l} u \longrightarrow du = \frac{dx}{x} \\ dv = x^3 dx \to v = \frac{x^4}{4} \end{array}\right\}\ 0.5\%$$

$$I = \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\left(\ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{dx}{x}\right) \longrightarrow 1\%$$

$$= \frac{x^4}{4}(\ln x)^2 - \frac{x^4}{8}\ln x + \frac{1}{8}\int x^3 dx$$

$$= \boxed{\frac{x^4}{4}(\ln x)^2 - \frac{x^4}{8}\ln x + \frac{1}{32}x^4 + C} \longrightarrow 1\%$$

Question 5 5 %

$$\frac{10}{(x^2-9)(x^2+1)} = \underbrace{\frac{A}{x-3}}_{0.5\,\%} + \underbrace{\frac{B}{x+3}}_{0.5\,\%} + \underbrace{\frac{Cx+D}{x^2+1}}_{1\,\%}$$

$$10 = A(x+3)(x^2+1) + B(x-3)(x^2+1) + (Cx+D)(x^2-9)$$

$x=3$: $\quad 10 = A\cdot 6\cdot 10 + B\cdot 0 + (C\cdot 3+D)\cdot 0$

$$\Longrightarrow \boxed{A = \frac{1}{6}} \quad 0.5\%$$

$x=-3$: $\quad 10 = A\cdot 0 + B(-6)\cdot 10 + (C\cdot(-3)+D)\cdot 0$

$$\Longrightarrow \boxed{B = -\frac{1}{6}} \quad 0.5\%$$

$x=0$: $\quad 10 = \underbrace{A\cdot 3 + B(-3)}_{\frac{1}{6}\cdot 3 + \left(-\frac{1}{6}\right)(-3) = 1} + D(-9) \quad \Big\} \Longrightarrow \boxed{D = -1} \quad 0.5\%$

$x=1$: $\quad 10 = \underbrace{A\cdot 8 + B\cdot(-4)}_{\frac{1}{6}\cdot 8 + \left(-\frac{1}{6}\right)(-4) = 2} + (C+D)(-8) \quad \Big\} \Longrightarrow C + \underbrace{D}_{-1} = -1$

$$\boxed{C = 0} \quad 0.5\%$$

$$I = \int \left(\frac{1}{6}\cdot\frac{1}{x-3} + \left(-\frac{1}{6}\right)\frac{1}{x+3} + \frac{-1}{x^2+1}\right) dx$$

$$= \boxed{\underbrace{\frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3|}_{0.5\,\%} \underbrace{- \tan^{-1}x}_{0.5\,\%} + C}$$