MA4002 Midterm Exam Solutions 2001

1.(a) Evaluate the indefinite integral $\int \frac{3x+1}{\sqrt{x}}\,dx$. Answer: $2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$.

(b) Calculate the area between $y = e^{3x}$ and the x-axis for $0 \le x \le 1$.

Answer: $\int_0^1 e^{3x}\,dx = \frac{1}{3}(e^3 - 1)$.

(c) Express as a definite integral (but do not evaluate) the limit of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} \sin(c_i^2 + 1)\,\Delta x_i$, where P is the partition with $x_i = \frac{i\pi}{n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$.

Answer: When $i = 0$, $a = x_0 = 0$ and when $i = n$, $b = x_n = \pi$. So using FTC2, we get $\int_0^\pi \sin(x^2 + 1)\,dx$.

(d) Evaluate $\frac{d}{dx}\int_0^x \ln(\cos(\sqrt{t}))\,dt$. Answer: $\ln(\cos(\sqrt{x}))$, using FTC1.

(e) Find an upper bound for the error E_S in the Simpson's Rule approximation of the definite integral $\int_0^2 f(x)\,dx$, using 200 subintervals, given that $M_4 \equiv \max_{x\in[0,2]} \left|\frac{d^4}{dx^4} f(x)\right| < 360$.

Answer: $h = 2/200 = 0.01$, $b - a = 2$, so $E_S < (0.01)^4(2)(360)/180 = 4 \times 10^{-8}$.

2. Evaluate the indefinite integral $\int \frac{(\ln t)^2}{t}\,dt$.

Answer: Substitute $u = \ln t$ to get answer $\frac{(\ln t)^3}{3} + C$.

3. Find the average value of $x \cos x$ on the interval $[0, \pi]$.

Answer: $\bar{f} = \frac{1}{\pi}\int_0^\pi x \cos x\,dx = -\frac{2}{\pi}$, after using integration by parts with $u = x$ and $dv = \cos x\,dx$.

4. Evaluate the definite integral $\int_2^3 \frac{x}{x^2 - 4x + 5}\,dx$.

Answer: Completing the square gives $x^2 - 4x + 5 = (x-2)^2 + 1$, so we substitute $u = x - 2$ to get $\int_0^1 \frac{u+2}{u^2+1}\,du = \left(\frac{1}{2}\ln(u^2+1) + 2\tan^{-1} u\right)\Big|_0^1 = \frac{1}{2}\ln 2 + \frac{\pi}{2}$.

5. Perform a partial fraction expansion of $\frac{4x - 4}{x^2(x-2)}$.

Answer: Put this equal to $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$. Multiply through both sides by the denominator $x^2(x-2)$ and compare to get answer $\frac{-1}{x} + \frac{2}{x^2} + \frac{1}{x-2}$.