MA4002 Midterm Exam Solutions 2002

1.(a) Evaluate the indefinite integral $\displaystyle\int \frac{x^3-1}{x\sqrt{x}}\,dx$ Answer: $\frac{2}{5}x^{\frac{5}{2}} + 2x^{-\frac{1}{2}} + C$.

(b) Calculate the area between $y = \cos 2x$ and the x-axis for $0 \le x \le \frac{\pi}{4}$.

Answer: $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x\,dx = \frac{1}{2}$.

(c) Express as a definite integral (but do not evaluate) the limit of the Riemann sum $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \ln(\tan(c_i))\,\Delta x_i$, where P is the partition with $x_i = \dfrac{2i}{n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$.

Answer: When $i = 0$, $a = x_0 = 0$ and when $i = n$, $b = x_n = 2$. So using FTC2, we get $\displaystyle\int_0^2 \ln(\tan x)\,dx$.

(d) Evaluate $\displaystyle\frac{d}{dx}\int_x^1 \tan(\cosh(\sin t))\,dt$. Interchange limits. Answer: $-\tan(\cosh(\sin x))$, using FTC1.

(e) Find an upper bound for the error E_T in the Trapezoidal Rule approximation of the definite integral $\displaystyle\int_0^3 f(x)\,dx$, using 150 subintervals, given that $M_2 \equiv \displaystyle\max_{x\in[0,3]}\left|\frac{d^2}{dx^2}f(x)\right| < 10$.

Answer: $h = 3/150 = 0.02$, $b - a = 3$, so $E_T < (0.02)^2(3)(10)/12 = 0.001$.

2. Evaluate the indefinite integral $\displaystyle\int \frac{3t^2}{t^6+1}\,dt$.

Answer: Substitute $u = t^3$ to get answer $\tan^{-1} t^3 + C$.

3. Find the average value of $x^2 \ln x$ on the interval $[1, e]$.

Answer: $\bar{f} = \dfrac{1}{e-1}\displaystyle\int_1^e x^2 \ln x\,dx = \frac{2e^3+1}{9(e-1)}$, after using integration by parts with $u = \ln x$ and $dv = x^2\,dx$.

4. Evaluate the definite integral $\displaystyle\int_0^2 \frac{x}{x^2+6x+9}\,dx$.

Answer: Note $x^2+6x+9 = (x+3)^2$, so we substitute $u = x+3$ to get $\displaystyle\int_0^1 \frac{u-3}{u^2}\,du = \ln 5 - \ln 3 - \frac{2}{5}$.

5. Perform a partial fraction expansion of $\dfrac{3x-5}{(x-1)(x^2-2x+2)}$.

Answer: Put this equal to $\dfrac{A}{x-1} + \dfrac{Bx+C}{x^2-2x+2}$. Multiply through both sides by the denominator $(x-1)(x^2-2x+2)$ and compare to get answer $\dfrac{-2}{x-1} + \dfrac{2x+1}{x^2-2x+2}$.