MA4002 Midterm Exam Solutions 2003

1.(a) Evaluate the indefinite integral $\int \frac{8x^2+2}{x^{\frac{1}{3}}}\,dx$ Answer: $3x^{\frac{8}{3}} + 3x^{\frac{2}{3}} + C$.

(b) Calculate the area between $y = \sin 3x$ and the x-axis for $0 \le x \le \frac{\pi}{3}$. Answer: $\int_0^{\frac{\pi}{3}} \sin 3x\,dx = \frac{2}{3}$.

(c) Express as a definite integral (but do not evaluate) the limit of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} \cos(e^{c_i} + c_i^2)\,\Delta x_i$, where P is the partition with $x_i = \frac{i\pi}{2n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$.
Answer: When $i = 0$, $a = x_0 = 0$ and when $i = n$, $b = x_n = \frac{\pi}{2}$. So using FTC2, we get $\int_0^{\frac{\pi}{2}} \cos(e^x + x^2)\,dx$.

(d) Evaluate $\frac{d}{dx}\int_0^{2x} \sin(\cos(\sin t))\,dt$. Answer: $2\sin(\cos(\sin 2x))$, using FTC1.

(e) Find an upper bound for the error E_S in the Simpson's Rule approximation of the definite integral $\int_2^4 f(x)\,dx$, using 200 subintervals, given that $M_4 \equiv \max_{x\in[2,4]} \left|\frac{d^4}{dx^4} f(x)\right| < 54$.
Answer: $h = (4-2)/200 = 0.01$, $b - a = 2$, so $E_S < (0.01)^4(2)(54)/180 = 6 \times 10^{-9}$.

2. Evaluate the indefinite integral $\int \sin^3 t \cos t\,dt$.
Answer: Substitute $u = \sin t$ to get answer $\frac{1}{4}\sin^4 t + C$.

3. Find the average value of xe^{2x} on the interval $[0, 1]$.
Answer: $\bar{f} = \frac{1}{1-0}\int_0^1 x\,e^{2x}\,dx = \frac{e^2+1}{4}$, after using integration by parts with $u = x$ and $dv = e^{2x}\,dx$.

4. Evaluate the definite integral $\int_2^3 \frac{x-7}{x^2-5x+4}\,dx$.
Answer: Note $x^2 - 5x + 4 = (x-1)(x-4)$, so we do a partial fraction expansion of the form $\frac{A}{x-1} + \frac{B}{x-4}$ to get $\int_2^3 \frac{2}{x-1} - \frac{1}{x-4}\,dx = (2\ln|x-1| - \ln|x-4|)\Big|_{x=2}^{x=3} = -3\ln 2$.

5. Perform a partial fraction expansion of $\frac{4}{(x-1)^2(x+1)}$.
Answer: Put this equal to $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$. Multiply through both sides by the denominator $(x-1)^2(x+1)$ and compare to get answer $\frac{-1}{x-1} + \frac{2}{(x-1)^2} + \frac{1}{x+1}$.