Question 1(a)

2%

$$I=\int x\, e^{-x^2}\cdot dx$$

$$\underbrace{u=x^2}_{0.5\%} \Longrightarrow du = 2x\cdot dx$$

$$I=\underbrace{\int e^{-u}\cdot \tfrac{1}{2}du}_{0.5\%} = \left.-\tfrac{1}{2}e^{-u}+c\right\}\ 0.5\%$$

$$=\boxed{-\tfrac{1}{2}e^{-x^2}+c}\quad 0.5\%$$

Question 1(b)

1%

$$A=\int_0^2 x\, e^{-x^2}\cdot dx \qquad 0.5\%$$

$$=-\tfrac{1}{2}e^{-x^2}\Big|_0^2=\boxed{\tfrac{1}{2}\left(1-e^{-4}\right)}$$

0.5%

Question 1(c)

3%

$$x_i=\frac{2i}{n}\Longrightarrow x_0=0,\ \dots\ x_n=2$$

$$\Delta x=\frac{2}{n}\Longrightarrow \frac{1}{n}=\frac{\Delta x}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{2}\sum_{i=1}^{n}5^{1+x}\cdot\Delta x\right)$$

$$=\underbrace{\frac{1}{2}}_{0.5\%}\int_0^2 \underbrace{5^{1+x}}_{0.5\%}\cdot dx=\frac{1}{2}\cdot\underbrace{\frac{5^{1+x}}{\ln 5}}_{0.5\%}\Bigg|_0^2$$

$$=\frac{1}{2}\cdot\frac{1}{\ln 5}\left(5^3-5\right)=\boxed{\frac{60}{\ln 5}}\quad 1\%$$

0.5% for correct limits of integration

Question 1(d)

2%

$$\frac{d}{dx}\left(\int_{x}^{x^2+x} \cos\sqrt{t+8}\cdot dt\right)$$

$$= (x^2+x)'\cdot \cos\sqrt{x^2+x+8} - (x)'\cos\sqrt{x+8}$$

$$= \boxed{\underbrace{(2x+1)}_{0.5\%}\ \underbrace{\cos\sqrt{x^2+x+8}}_{0.5\%}\ \underbrace{-}_{0.5\%}\ \underbrace{\cos\sqrt{x+8}}_{0.5\%}}$$

Question 1(e)

3%

$\cos x$, $\cos^2 x$ are even; 0.5%

$\tan x$, $\tan(x^5)$ are odd. 0.5%

$$I = \int_{-\pi/4}^{\pi/4}\left(\cos^2 x + \underbrace{\tan(x^5)}_{=0 \;\; 0.5\%}\right) dx = \int_{-\pi/4}^{\pi/4} \cos^2 x \cdot dx$$

Approach A

$$I = 2\int_0^{\pi/4} \cos^2 x\cdot dx$$

Approach B: proceed with $\int_{-\pi/4}^{\pi/4}$

$$= 2\int_0^{\pi/4} \underbrace{\frac{1+\cos(2x)}{2}}_{0.5\%}\, dx = \underbrace{\left(x + \frac{\sin(2x)}{2}\right)}_{0.5\%}\Bigg|_0^{\pi/4} = \boxed{\frac{\pi}{4} + \frac{1}{2}} \quad 0.5\%$$

Question 2

4%

$$I = \int \sin^2 x \cdot \sin x \cdot dx$$

$u = \cos x$ 0.5%

$du = -\sin x \cdot dx$ 0.5%

$\sin^2 x = 1 - \cos^2 x = 1 - u^2$ 0.5%

$$I = \underbrace{\int (1-u^2)(-du)}_{1\%} = \underbrace{\left(-u + \frac{u^3}{3}\right)}_{0.5\%\text{ each term}} + C$$

$$= \underbrace{-\cos x + \frac{\cos^3 x}{3} + C}_{0.5\%}$$

Question 3

4%

$$\bar{f} = \frac{1}{2}\int_0^2 \frac{dx}{x^2+6x+10} \Big\} \ 0.5\%$$

$$x^2+6x+10 = \underbrace{(x+3)^2+1}_{0.5\%} = u^2+1$$

$$\underbrace{u = x+3}_{0.5\%}, \quad du = dx$$

$$\underbrace{\bar{f} = \frac{1}{2}\int_{u=3}^{u=5} \frac{du}{u^2+1}}_{1\%} = \frac{1}{2}\underbrace{\tan^{-1}(u)}_{0.5\%}\Big|_{u=3}^{u=5}$$

$$= \boxed{\frac{1}{2}\left(\tan^{-1}5 - \tan^{-1}3\right)} \ 1\%$$

....

Question 4

5%

$$I = \int x^8 \underbrace{\ln^2 x}_{u} \cdot du$$

0.5% $u \to du = \frac{2\ln x}{x}dx$

$v = \frac{x^9}{9}$ } 0.5%

$$I = \ln^2 x \cdot \frac{x^9}{9} - \int \frac{x^9}{9}\cdot\frac{2\ln x}{x}dx$$

$$= \frac{x^9\cdot\ln^2 x}{9} - \frac{2}{9}\boxed{\int x^8 \cdot \underbrace{\ln x}_{u} \cdot dx}$$

} 1%

} 2% for first int-n by parts

similar 2%

$$= \frac{x^9\cdot\ln^2 x}{9} - \frac{2}{9}\left[\frac{x^9}{9}\cdot\ln x - \int\frac{x^9}{9}\cdot\frac{1}{x}dx\right]$$

$$= \boxed{\frac{x^9\cdot\ln^2 x}{9} - \frac{2}{81}x^9\cdot\ln x + \frac{2}{729}x^9 + C} \ 1\%$$

Question 5

6 %

$$\frac{3x^2+5}{(x-1)^3(x+1)} =$$

$$= \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$$

2% "
0.5% each correct term

$$3x^2+5 = A\cdot(x-1)^3 + B\cdot(x+1)(x-1)^2 + C(x+1)\cdot(x-1) + D(x+1)$$

$x=1$: $\quad 8 = D\cdot 2 \longrightarrow \boxed{D=4}$

0.5% each correct coef-t

$x=-1$: $\quad 8 = A\cdot(-8) \longrightarrow \boxed{A=-1}$

$x=0$: $\quad 5 = \underbrace{A}_{=-1}(-1) + B\cdot 1 + C\cdot(-1) + \underbrace{D}_{=4}\cdot 1$

$$\boxed{B-C=0}$$

$x=2$: $\quad \underbrace{3\cdot 4+5}_{=17} = \underbrace{A\cdot 1}_{=-1} + \underbrace{B\cdot 3 + \underset{=B}{C}\cdot 3}_{=6B} + \underbrace{D\cdot 3}_{=12}$

$6B=6$, $\quad \boxed{B=C=1}$

$$\boxed{-\frac{1}{x+1} + \frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{4}{(x-1)^3}}$$

$$\boxed{I = -\ln|x+1| + \ln|x-1| - \frac{1}{x-1} - \frac{2}{(x-1)^2} + C}$$

0.5% for each correct term