Question 1(a)

2%

$$\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \int e^{-u}\cdot 2\cdot du \quad 0.5$$

0.5 $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= -2e^{-u} + C \quad 0.5 \text{ else}$$

$$= \boxed{-2e^{-\sqrt{x}} + C} \quad 0.5$$

Question 1(b)

2%

$$\underbrace{A = \int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx}_{0.5} = \underbrace{-2e^{-\sqrt{x}}\Big|_0^\infty}_{0.5}$$

$$= -2e^{-\infty} + 2e^{-0}$$

$$= \underbrace{-0}_{0.5} + \underbrace{2}_{0.5} = \boxed{2}$$

Question 1(c)

2%

I

0.5 $\left\{ x_i = \frac{i}{n},\ x_0 = 0,\ x_n = 1;\quad \Delta x = \frac{1}{n} \right.$

$$\frac{1}{n}\sum_{i=1}^{n} \cos\left(\frac{3(i-1)}{n}\right) = \sum_{i=1}^{n} \cos(3x_{i-1})\cdot\Delta x$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\sum\cdots\right)$$

$$= \int_0^1 \cos(3x)\cdot dx \quad 0.5$$

$$= \underbrace{\frac{1}{3}\sin(3x)\Big|_0^1}_{0.5} = \boxed{\frac{\sin 3}{3}} \quad 0.5$$

II

$x_i = \frac{3i}{n} \Rightarrow x_0 = 0,\ x_n = 3$

$\Delta x = \frac{3}{n},\quad \frac{\Delta x}{3} = \frac{1}{n}$ 0.5

$$\frac{1}{n}\sum_{i=1}^{n} \cos\left(3\frac{(i-1)}{n}\right) = \sum_{i=1}^{n} \cos(x_{i-1})\cdot\frac{\Delta x}{3}$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\sum\cdots\right) = \int_0^3 \cos x\cdot\frac{dx}{3} \quad 0.5$$

$$= \underbrace{\frac{\sin x}{3}\Big|_0^3}_{0.5} = \underbrace{\boxed{\frac{\sin 3}{3}}}_{0.5}$$

Question 1(d)

$$\frac{d}{dx}\left(\int_{\sin x}^{\sin(3x)} \sqrt{1+\sqrt{t}}\cdot dt\right) =$$

$$= \underbrace{3\cdot\cos(3x)}_{0.5}\cdot\underbrace{\sqrt{1+\sqrt{\sin(3x)}}}_{0.5}\ \underbrace{-\cos x}_{\substack{0.5\\(\text{incl. "}-\text{"})}}\cdot\underbrace{\sqrt{1+\sqrt{\sin x}}}_{0.5}$$

2%

Question 1(e)

2%

$$\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\cdot dx}_{\shortparallel} + \underbrace{\int_{-\frac{\pi}{2}}^{\pi/2} \sin(x^3)\cdot dx}_{\shortparallel\ 0.5}$$

0 as $\sin(x^3)$ is odd

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1-\cos(2x)}{2}\,dx \quad 0.5\%$$

$$\shortparallel$$

$$\frac{x}{2}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{\sin(2x)}{4}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$$

0.5% 0.5%

Question 2

5%

$$I = \int \sin^5 x\cdot dx = \int \underbrace{(\sin^2 x)^2}_{\substack{\shortparallel\\(1-\cos^2 x)^2}}\cdot \sin x\cdot dx$$

1%

$$\left.\begin{aligned} u &= \cos x\\ -du &= \sin x\cdot dx\end{aligned}\right\} \Longrightarrow \left. I = -\int (1-u^2)^2\cdot du \right\} \ 1.5\%$$

$$= \int(-1+2u^2-u^4)\cdot du \quad 0.5\%$$

$$= -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C \quad 1\%$$

$$= \boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C} \quad 1\%$$

4%

Question 3

$$\bar{f} = \frac{1}{4}\int_0^4 \frac{2}{x^2+4x+3} \quad 1\%$$

I $x^2+4x+3 = (x+2)^2 - 1$

$u = x+2$

$$\bar{f} = \frac{1}{4}\int_{u=2}^{u=6} \frac{2}{u^2-1}\,du \quad 1\%$$

$$= \frac{1}{4}\cdot 2\cdot\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\Bigg|_{u=2}^{u=6} \quad 1\%$$

$$= \frac{1}{4}\left(\ln\frac{5}{7} - \ln\frac{1}{3}\right) \quad 1\%$$

$$= \boxed{\frac{1}{4}(\ln 5 - \ln 7 + \ln 3)}$$

II $x^2+4x+3 = (x+3)(x+1)$

$$\frac{2}{x^2+4x+3} = \frac{A}{x+1} + \frac{B}{x+3} \quad 1\%$$

$2 = A(x+3) + B(x+1)$ 0.5 each

$x=-3 \to 2 = B\cdot(-2) \to B=-1$

$x=-1 \to 2 = A\cdot 2 \to A=1$

$$\bar{f} = \frac{1}{4}\int_0^4\left(\frac{1}{x+1} - \frac{1}{x+3}\right)dx$$

$$= \left(\frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x+3|\right)\Big|_0^4 \quad 1\%$$

5%

Question 4

$$I \quad \int \underbrace{x^2}_{u}\cdot\underbrace{\cos(2x)\cdot dx}_{dv}$$

$du = 2x\cdot dx$ $\quad v = \frac{\sin(2x)}{2}$ 0.5%

$$I = x^2\cdot\frac{\sin(2x)}{2} - \int \underbrace{x}_{u}\cdot\underbrace{\sin(2x)\cdot dx}_{dv}$$ 1.5 with partial marks

$du = dx$ $\quad v = \frac{-\cos(2x)}{2}$ 0.5

$$I = x^2\cdot\frac{\sin(2x)}{2} - \left(x\cdot\left(\frac{-\cos(2x)}{2}\right) - \int\left(\frac{-\cos(2x)}{2}\right)\cdot dx\right) \quad 1.5$$

$$= \boxed{\frac{x^2\cdot\sin(2x)}{2} + \frac{x\cos(2x)}{2} \cancel{+} \underbrace{\frac{\sin(2x)}{4}}_{1\%} + C}$$

Question 5

6%

$$(x^2-2x+1)(x^2+1) = (x-1)^2(x^2+1)$$

$$\frac{2-4x}{(x^2-2x+1)(x^2+1)} = \frac{A}{\underbrace{x-1}_{0.5}} + \frac{B}{\underbrace{(x-1)^2}_{0.5}} + \frac{Cx+D}{\underbrace{x^2+1}_{1}} \quad \Sigma = 2\%$$

$$2-4x = \underbrace{A(x-1)\cdot(x^2+1)}_{A(x^3+x-x^2-1)} + B(x^2+1) + \underbrace{(Cx+D)(x^2-2x+1)}_{C(x^3-2x^2+x)+D(x^2-2x+1)}$$

$$2-4x = x^3(A+C) + x^2(-A+B-2C+D) + x(A+C-2D) + (-A+B+D)$$

Also, set $x=1 \to -2 = B\cdot 2 \to \boxed{B=-1}$ 0.5

$A+C=0$

$-A+B-2C+D=0$

$\underbrace{A+C}_{0}-2D=-4 \longrightarrow \boxed{D=2}$ 0.5

$-A+B+D=2 \longrightarrow \boxed{A=B+D-2=-1}$ 0.5 $\to \boxed{C=-A=1}$ 0.5

$$\boxed{\frac{-1}{x-1}+\frac{-1}{(x-1)^2}+\frac{x+2}{x^2+1}}$$

i.e. each correct coefficient $\to 0.5\%$

$$I = -\ln|x-1| + \frac{1}{x-1} + \int\frac{x}{x^2+1}dx + 2\int\frac{dx}{x^2+1}$$

$$= \boxed{-\ln|x-1| + \frac{1}{x-1} + \frac{1}{2}\ln(x^2+1) + 2\tan^{-1}x + C}$$

each correct term $\to 0.5\%$