Question 1(a)

2%

$$I = \int x \cdot \cos(x^2+1) \cdot dx$$

$$\left.\begin{aligned} &\boxed{u = x^2+1} \ \tfrac{1}{2}\% \\ &du = 2x \cdot dx \\ &\tfrac{1}{2} du = x \cdot dx \end{aligned}\right\} \rightarrow I = \int \cos u \cdot \frac{du}{2} \ \Big\} \tfrac{1}{2}\%$$

$$= \frac{\sin u}{2} + C \ \Big\} \tfrac{1}{2}\%$$

$$= \boxed{\frac{\sin(x^2+1)}{2} + C} \ \tfrac{1}{2}\%$$

Question 1(b)

2%

$$A = \int_0^{1/2} \underbrace{x \cdot \cos(x^2+1)}_{\geq 0} \cdot dx \ \Big\} \ 0.5\%$$

$$= \frac{1}{2} \sin(x^2+1) \Big|_0^{1/2} \ \Big\} \ 0.5\%$$

$$= \boxed{\underbrace{\frac{1}{2} \sin\left(\frac{5}{4}\right)}_{0.5\%} - \underbrace{\frac{1}{2} \sin(1)}_{0.5\%}}$$

("–" should be correct)

Question 1(c)

2%

$$S = \lim_{n \to \infty} \left(\frac{1}{n} \sum_{i=1}^{n} \sin\left(\frac{i-1}{4n}\right)\right)$$

$$x_{i-1} = \frac{i-1}{4n} \longrightarrow \quad i = 1: \quad x_0 = 0$$

$$i = n: \quad x_n = \frac{n}{4n} = \frac{1}{4}$$

$$x_i = \frac{i}{4n}$$

$$\Delta x = x_i - x_{i-1} = \frac{1}{4n}$$

$$4 \Delta x = \frac{1}{n}$$

$$S = \lim_{n \to \infty} \left(4 \Delta x \sum_{i=1}^{n} \sin(x_{i-1}) \right) = 4 \int_0^{1/4} \sin x \cdot dx$$

partial marks 1%

Alternative Sol-n 1%

$$S = \int_0^1 \sin\left(\frac{x}{4}\right) \cdot dx$$

$$= -4 \cos x \Big|_0^{1/4} = \boxed{4 - 4 \cos \frac{1}{4}} \ 0.5\%$$

0.5%

2 %
Question 1(d)

$$\frac{d}{dx}\left(\int_x^{x\cdot\sin(x^3)} \ln(1+t)\cdot dt\right) =$$

$$= \underbrace{\left(x\cdot\sin(x^3)\right)'}_{\sin(x^3)+x\cdot\cos(x^3)\cdot 3x^2}\cdot\ln\left(1+x\cdot\sin(x^3)\right) - \ln(1+x) =$$

$$= \Big[\underbrace{\sin(x^3)}_{0.5\%} + \underbrace{3x^3\cdot\cos(x^3)}_{0.5\%}\Big]\cdot\overbrace{\ln\left(1+x\cdot\sin(x^3)\right)}^{0.5\%} - \underbrace{- \ln(1+x)}_{0.5\%\ (\text{with "}-\text{"})}$$

Question 1(e)

4 %

$$I = \underbrace{\int_{-\pi/2}^{\pi/2} \underbrace{x\cdot\sin(x^2)}_{odd}\,dx}_{=0} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{x^2}_{u}\cdot\underbrace{\cos x\cdot dx}_{dv}\quad 0.5\%$$

$$du = 2x\cdot dx \qquad v = \sin x$$

$$I = \underset{1\%}{0} + \underbrace{x^2\cdot\sin x\Big|_{-\frac{\pi}{2}}^{\pi/2}}_{\left(\frac{\pi}{2}\right)^2-\left(-\left(\frac{\pi}{2}\right)^2\right)} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\cdot 2x\cdot dx \quad 0.5\%$$

$u = x$, $dv = \sin x\cdot dx$, $v = -\cos x$ 0.5%

$$I = \frac{\pi^2}{2} - 2\Big(\underbrace{x\cdot(-\cos x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{0} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(-\cos x)\cdot dx\Big) = \frac{\pi^2}{2} - 2\sin x\Big|_{-\frac{\pi}{2}}^{\pi/2}$$

0.5% 0.3

$$= \boxed{\frac{\pi^2}{2} - 4}$$

0.5%

Question 2

4%

$$I = \int (\sin x)^2\cdot(\cos x)^3\cdot dx$$

$$I = \int (\sin x)^2(\cos x)^2\cdot\cos x\cdot dx$$

1%

$u = \sin x$, $du = \cos x\cdot dx$

$(\cos x)^2 = 1 - u^2$

$$I = \underbrace{\int u^2(1-u^2)\cdot du}_{1\%} = \underbrace{\frac{u^3}{3} - \frac{u^5}{5} + C}_{0.5+0.5\%}$$

$$= \boxed{\frac{1}{3}(\sin x)^3 - \frac{1}{5}(\sin x)^5 + C}$$

0.5 + 0.5 %

Question 3 5%

$$\bar{f} = \frac{1}{2}\int_0^2 \frac{2x}{x^2-4x+5}\cdot dx \quad \Big\}\ 0.5\%$$

$$x^2-4x+5 = (x-2)^2+1 \longrightarrow u = x-2, \quad du = dx \quad 0.5\%$$

$$\bar{f} = \frac{1}{2}\int_{u=-2}^{u=0} \frac{2\cdot(u+2)}{u^2+1}\,du \quad \Big\}\ 1\%$$

$$= \underbrace{\int_{-2}^{0} \frac{u}{u^2+1}\,du}_{0.5\%} + \underbrace{2\int_{-2}^{0}\frac{1}{u^2+1}\,du}_{0.5\%}$$

$$= \underbrace{\overbrace{\frac{1}{2}\ln(u^2+1)}^{0.5}\Big|_{-2}^{0}}_{\underbrace{\frac{1}{2}(0-\ln 5)}_{0.5}} + \underbrace{\overbrace{2\cdot\tan^{-1}(u)}^{0.5}\Big|_{-2}^{0}}_{\underbrace{2(0-\tan^{-1}(-2))}_{0.5}} = \boxed{2\tan^{-1}(2) - \frac{\ln 5}{2}}$$

Question 4 3%

$$I = \int \underbrace{\tan^{-1}x}_{u}\cdot \underbrace{dx}_{dv} \longrightarrow v = x$$

$$0.5\% \quad u \rightarrow du = \frac{1}{x^2+1}\,dx \quad 0.5\%$$

$$I = (\tan^{-1}x)\cdot x - \int\frac{x}{x^2+1}\,dx \quad \Big\}\ 1\%$$

$$= \boxed{x\cdot\tan^{-1}x - \frac{1}{2}\ln(x^2+1) + C} \quad 1\%$$

Question 5

6%

$$f = \frac{3-x}{(x^2-3x+2)(x^2+1)}$$

$$= \frac{3-x}{(x-1)(x-2)(x^2+1)}$$

$$\boxed{f = \underbrace{\frac{A}{x-1}}_{0.5} + \underbrace{\frac{B}{x-2}}_{0.5} + \underbrace{\frac{Cx+D}{x^2+1}}_{1\%}}$$

$$3-x = A(x-2)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x-2)$$

set $x=1$: $2 = A(-1)\cdot(2) + B\cdot 0 + (Cx+D)0 \Rightarrow \boxed{A=-1}$ 0.5

set $x=2$: $1 = A\cdot 0 + B\cdot(1)\cdot(5) + (Cx+D)\cdot 0 \Rightarrow \boxed{B=\frac{1}{5}}$ 0.5

$$3-x = x^3\underbrace{(A+B+C)}_{=0} + x^2(-2A-B-3C+D) + x(A+B+2C-3D) + \underbrace{(-2A-B+2D)}_{=3}$$

$\boxed{C = -(A+B) = \frac{4}{5}}$ 0.5

$2D = 3 + 2A + B =$ 0.5

$2D = \frac{6}{5} \Rightarrow \boxed{D = \frac{3}{5}}$

$$I = \int f\cdot dx = \int \frac{(-1)}{x-1}dx + \int \frac{1/5}{x-2}dx + \frac{4}{5}\int \frac{x}{x^2+1}dx + \frac{3}{5}\int \frac{1}{x^2+1}dx$$

$$I = \boxed{\underbrace{-\ln|x-1|}_{0.5} + \underbrace{\frac{1}{5}\ln|x-2|}_{0.5} + \underbrace{\frac{2}{5}\ln(x^2+1)}_{0.5} + \underbrace{\frac{3}{5}\tan^{-1}x}_{0.5} + C}$$