Question 1(a) $I = \int \frac{\sin(\ln x)}{x} \cdot dx$ (2%)

0.5% $\begin{cases} u = \ln x \\ du = \frac{1}{x} dx \end{cases} \longrightarrow I = \int \sin(u) \cdot du$ 0.5%

$= -\cos(u) + C$ 0.5%.

$= \boxed{-\cos(\ln x) + C}$ 0.5%.

.....

Question 1(b) (2%)

Area $0 \le \ln x \le \ln 2 < \ln e = 1$

$\Rightarrow \frac{\sin(\ln x)}{x} \ge 0$ for $1 \le x \le 2$

$\text{Area} = \int_1^2 \frac{\sin(\ln x)}{x} dx$ 0.5%

$= -\cos(\ln x)\Big|_1^2 = -\cos(\ln 2) + \cos(\ln 1)$ 0.5%.

(0.5% for $-\cos(\ln x)\Big|_1^2$; $\cos(\ln 1) = \cos 0 = 1$)

$= \boxed{1 - \cos(\ln 2)}$ 0.5%.

.....

Question 1(c) (3%)

$\lim_{n \to \infty} S_n$ where $S_n = \sum_{i=1}^{n} \frac{1}{i+n}$

S On $(0,1)$, let $x_i = \frac{i}{n}$, $\Delta x = \frac{1}{n}$ 0.5%

$i = 0, \dots, n$

$\Rightarrow S_n = \sum_{i=1}^{n} \frac{1}{\left(\frac{i}{n} + 1\right) n} = \sum_{i=1}^{n} \frac{1}{x_i + 1} \cdot \Delta x$ 1%

Right R. Sum for $\frac{1}{x+1}$

$\lim_{n \to \infty} S_n = \int_0^1 \frac{dx}{x+1}$ 0.5%.

$= \ln(x+1)\Big|_0^1 = \ln 2 - \ln 1 = \boxed{\ln 2}$ 0.5%

Question 1(d) $\frac{d}{dx}\left(\int_{x^2}^{x+x^2} \cos(1+t^2)\,dt\right) =$ 2 %.

$= \underbrace{(1+2x)}_{0.5\%} \cdot \overbrace{\cos\left(1+\underbrace{x^2+2x^3+x^4}_{\text{or } (x+x^2)^2}\right)}^{0.5\%.}$

$\underbrace{-2x}_{0.5\%.} \cdot \underbrace{\cos(1+x^4)}_{\text{incl. "}-\text{" cost 0.25\%}}$ 0.5 %.

Question 1(e)

3 %.

Question 2 $I = \int (\sin x)^2 (\cos x)^8 \sin x \cdot dx$

$u = \cos x$ } 0.5 %.
$du = -\sin x \cdot dx$

$(\sin x)^2 \to (1-u^2)$ $(\cos x)^8 \to u^8$ $\sin x \cdot dx \to -du$

$I = \int \underbrace{(1-u^2)}_{0.5\%.} \underbrace{u^8}_{0.5\%.} \underbrace{(-du)}_{0.5\%.}$

$= \int (u^{10} - u^8)\,du = \frac{u^{11}}{11} - \frac{u^9}{9} + C$ } 0.5 %.

$= \boxed{\frac{(\cos x)^{11}}{11} - \frac{(\cos x)^9}{9} + C}$ 0.5 %.

Question 3 (4%)

$$\bar{f} = \frac{1}{2}\int_{-1}^{1} \frac{x+3}{(x^2+2x+5)}\cdot dx \qquad (0.5\%)$$

$x^2+2x+5 = (x+1)^2+4 \rightarrow u = x+1$ } 0.5

$du = dx$

$$\bar{f} = \frac{1}{2}\int_{u=0}^{u=2} \frac{u+2}{u^2+4}\,du \quad \} 1\%$$

$$= \frac{1}{2}\int_{u=0}^{u=2}\left(\frac{u}{u^2+4} + \frac{2}{u^2+2^2}\right)du$$

$$= \frac{1}{2}\left(\underbrace{\frac{1}{2}\ln(u^2+4)}_{0.5} + \underbrace{\tan^{-1}\left(\frac{u}{2}\right)}_{0.5}\right)\Big|_0^2 = \underbrace{\boxed{\frac{1}{4}\ln 2 + \frac{\pi}{8}}}_{0.5\ \ 0.5}$$

(where we used $\ln 8 - \ln 4 = \ln 2$, $\tan^{-1}1 = \frac{\pi}{4}$, $\tan^{-1}0 = 0$)

Question 4 (3%)

$$I = \int x \cdot \underbrace{\tan^{-1}x}_{u} \cdot dx$$

0.5% $\left.\begin{array}{l} u = \tan^{-1}x \\ dv = x\cdot dx \end{array}\right\} \rightarrow$ $\left.\begin{array}{l} du = \frac{1}{x^2+1}\,dx \\ v = \frac{x^2}{2} \end{array}\right\}$ 0.5

$$I = \left(\tan^{-1}x\right)\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x^2+1}\cdot dx \quad \} 0.5$$

$$= \frac{x^2}{2}\cdot\tan^{-1}x - \frac{1}{2}\int\frac{(x^2+1)-1}{x^2+1}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\underbrace{\left(1 - \frac{1}{x^2+1}\right)}_{0.5}dx$$

$$= \boxed{\frac{x^2}{2}\tan^{-1}x - \underbrace{\frac{x}{2}}_{0.5} + \underbrace{\frac{1}{2}\tan^{-1}x}_{0.5} + C}$$

Question 5 6 %

$$f(x) = \frac{4 - 3x}{(x^2 - x - 6)(x^2 - 4)}$$

$$x^2 - x - 6 = (x-3)(x+2)$$

$$x^2 - 4 = (x-2)(x+2)$$

$$f(x) = \frac{4 - 3x}{(x-3)(x+2)^2(x-2)}$$

$$= \underbrace{\frac{A}{x-3}}_{0.5} + \underbrace{\frac{B}{x+2}}_{0.5} + \underbrace{\frac{C}{(x+2)^2}}_{0.5} + \underbrace{\frac{D}{(x-2)}}_{0.5} \Big\} \ 2\%$$

$$4 - 3x = A(x+2)^2(x-2) + B(x-3)(x+2)(x-2) + C(x-3)(x-2) + D(x-3)(x+2)^2$$

$x = 3$: $-5 = A \cdot 5^2 \cdot 1 + B \cdot 0 + C \cdot 0 \cdot D \cdot 0$

$\boxed{A = -\frac{1}{5}}$ 0.5 %

✓ method is OK

$x = -2$: $10 = A \cdot 0 \cdot B \cdot 0 \cdot C(-5)(-4) + D \cdot 0$

$\boxed{C = \frac{1}{2}}$ 0.5 %

$x = 2$: $-2 = A \cdot 0 + B \cdot 0 \cdot C \cdot 0 + D(-1)4^2$

$\boxed{D = \frac{1}{8}}$ 0.5 %

$x = 0$: $4 = \underbrace{A}_{-\frac{1}{5}} \cdot 2^2(2) + B(-3) \cdot 2 \cdot (-2) + \underbrace{C}_{\frac{1}{2}} \cdot 6 + \underbrace{D}_{\frac{1}{8}} \cdot (-3) \cdot 2^2$

$4 = +\frac{8}{5} + 12B + 3 - \frac{3}{2} \Rightarrow 40 = +16 + 120 \cdot B + 15$ 0.5%.

$9 = 120B \rightarrow \boxed{B = \frac{3}{40}}$

0.5 % each correct term

$$I = \underbrace{-\frac{1}{5} \cdot \ln|x-3|}_{0.5} + \underbrace{\frac{3}{40}\ln|x+2|}_{0.5} \underbrace{- \frac{1}{2(x+2)}}_{0.5} + \underbrace{\frac{1}{8}\ln|x-2|}_{0.5} + C \quad 2\%$$